



Study Prep

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MASTER TABLE: TRIG IDENTITIES

◆ **NOTE:** This table spans multiple videos.

TRIG IDENTITIES			
Name	Identity	Example	Use when...
Reciprocal	$\csc \theta = \frac{1}{\sin \theta}$	$\sec \frac{\pi}{3} = \frac{1}{\cos \frac{\pi}{3}} = \frac{1}{\left(\frac{1}{2}\right)} = 2$	You need to rewrite an expression in terms of sin & cos
	$\sec \theta = \frac{1}{\cos \theta}$		
	$\cot \theta = \frac{1}{\tan \theta}$		
Quotient	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\tan \frac{\pi}{4} = \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)}$	
	$\cot \theta = \frac{\cos \theta}{\sin \theta}$		
Even – Odd	$\cos(-\theta) = \underline{\hspace{1cm}} \cos \theta$	$\cos\left(-\frac{\pi}{4}\right) =$ $\csc\left(\frac{\pi}{6}\right) =$	<i>argument</i> is _____.
	$\sin(-\theta) = \underline{\hspace{1cm}} \sin \theta$		
	$\tan(-\theta) = \underline{\hspace{1cm}} \tan \theta$		
Pythagorean	$\sin^2 \theta + \cos^2 \theta = 1$	$\sin^2 \frac{11\pi}{6} + \cos^2 \frac{11\pi}{6} =$	you see trig functions _____.
	$\underline{\hspace{1cm}} \theta + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \theta$		
	$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} \theta = \underline{\hspace{1cm}} \theta$		
Sum & Diff.	$\sin(a \pm b) = \underline{\hspace{1cm}} a \underline{\hspace{1cm}} b \pm \underline{\hspace{1cm}} a \underline{\hspace{1cm}} b$	$\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) =$	argument contains a _____/_____ OR multiples of 15° or $\frac{\pi}{12}$
	$\cos(a \pm b) = \underline{\hspace{1cm}} a \underline{\hspace{1cm}} b \mp \underline{\hspace{1cm}} a \underline{\hspace{1cm}} b$		
	$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$		
Double Angle	$\sin 2\theta = 2 \underline{\hspace{2cm}}$	$\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12} =$	argument contains _____ OR you recognize a _____ of the identity
	$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= 1 - 2 \sin^2 \theta$ $= 2 \cos^2 \theta - 1$		
	$\tan 2\theta = \frac{2 \tan \theta}{1 - \underline{\hspace{1cm}}^2 \theta}$		



TOPIC: INTRODUCTION TO TRIG IDENTITIES

Even & Odd Identities

◆ If you know a function is **even** or **odd**, you can easily find $f(\text{_____})$.

Even Function	Odd Function
$f(x) = \cos x$ $f(-x) =$ [$f(x)$ $-f(x)$] Symmetric on [y - axis ORIGIN] Trig Functions: [$\cos$ $\sin$ $\tan$] [$\sec$ $\csc$ $\cot$]	$f(x) = \sin x$ $f(-x) =$ [$f(x)$ $-f(x)$] Symmetric on [y - axis ORIGIN] Trig Functions: [$\cos$ $\sin$ $\tan$] [$\sec$ $\csc$ $\cot$]

◆ An **identity** is an equation which is TRUE for _____ possible values.

TRIG IDENTITIES			
Name	Identity	Example	Use when...
Even - Odd	$\cos(-\theta) = \text{_____} \cos \theta$	$\cos\left(-\frac{\pi}{4}\right) =$	argument is _____.
	$\sin(-\theta) = \text{_____} \sin \theta$	$\csc\left(-\frac{\pi}{6}\right) =$	
	$\tan(-\theta) = \text{_____} \tan \theta$		

**TOPIC: INTRODUCTION TO TRIG IDENTITIES****EXAMPLE**

Use the even/odd identities to rewrite the expression with no negative arguments in terms of one trig function.

$$(A) \quad -\tan(-\theta)$$

$$(B) \quad \frac{\sin(-\theta)}{\cos(-\theta)}$$

Recall

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

*(Even – Odd Identities)***PRACTICE**

Use the even-odd identities to evaluate the expression.

$$(A) \quad \cos(-\theta) - \cos \theta$$

$$(B) \quad -\cot(\theta) \cdot \sin(-\theta)$$

TOPIC: INTRODUCTION TO TRIG IDENTITIES**PRACTICE**

Use the even-odd identities to evaluate the expression.

(A)

$$\sec\left(-\frac{4\pi}{5}\right)$$

$$\cos\frac{4\pi}{5}$$

$$-\cos\frac{4\pi}{5}$$

$$\sec\frac{4\pi}{5}$$

$$-\sec\frac{4\pi}{5}$$

Recall

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

*(Even - Odd Identities)***(B)**

$$\sin(-38^\circ)$$

$$\sin 38^\circ$$

$$-\sin 38^\circ$$

$$-\sin(-38^\circ)$$

$$\frac{1}{-\sin 38^\circ}$$

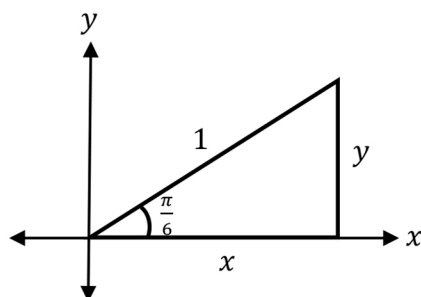


TOPIC: INTRODUCTION TO TRIG IDENTITIES

Pythagorean Identities

◆ You'll need the **Pythagorean Identities** to simplify expressions with _____ trig functions.

- These identities come from combining the Pythagorean Theorem with the Unit Circle.



$$a^2 + b^2 = c^2$$

$$y^2 + x^2 = 1$$

TRIG IDENTITIES			
Name	Identity	Example	Use when...
Pythagorean	$\sin^2 \theta + \cos^2 \theta = 1$	$\sin^2 \frac{11\pi}{6} + \cos^2 \frac{11\pi}{6} =$	you see trig functions squared.
	$\underline{\hspace{1cm}} \theta + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \theta$		
	$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} \theta = \underline{\hspace{1cm}} \theta$		

◆ To rewrite trig expressions, you'll need to recognize different _____ of the Pythagorean Identities.

EXAMPLE

Use the Pythagorean Identities to rewrite the expression as a single term.

(A)

$$\sec^2 \theta - \tan^2 \theta$$

(B)

$$(1 - \cos \theta)(1 + \cos \theta)$$

TOPIC: INTRODUCTION TO TRIG IDENTITIES**EXAMPLE**

Use the Pythagorean Identities to rewrite the expression as a single term.

$$\frac{1}{1 + \cos \theta}$$

Recall

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

*(Even – Odd Identities)***PRACTICE**

Use the Pythagorean Identities to rewrite the expression as a single term.

$$(1 + \csc \theta)(1 - \csc \theta)$$

PRACTICE

Use the Pythagorean Identities to rewrite the expression with no fraction.

$$\frac{1}{1 - \sec \theta}$$

**TOPIC: INTRODUCTION TO TRIG IDENTITIES****Simplifying Trig Expressions**

◆ You'll need to use ALL trig identities to *fully* simplify expressions.

EXAMPLE

Simplify the expression.

(A) $\tan(-\theta) \cdot \csc \theta$

(B) $\frac{\sin^2 \theta}{1 + \cos \theta}$

HOW TO: Fully Simplify Trig Expressions

Trig expressions are fully simplified when...

- ☐ all arguments are _____
- ☐ expression contains NO _____
- ☐ expression contains as few trig fcn's as possible

Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of _____ & _____
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall**Fundamental Trig Identities**Reciprocal & Quotient

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Even/Odd

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

Pythagorean

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

TOPIC: INTRODUCTION TO TRIG IDENTITIES

EXAMPLE

Simplify the expression.

(A)

$$\frac{\sin^2 \theta - \tan^2 \theta}{\sin \theta + \tan \theta}$$

(B)

$$\frac{\cos \theta + \csc \theta}{\cos \theta} + \frac{\sin \theta - \sec \theta}{\sin \theta}$$

HOW TO: Fully Simplify Trig Expressions

Trig expressions are fully simplified when...

- ☐ all arguments are positive
- ☐ expression contains NO fractions
- ☐ expression contains as few trig fcn's as possible

Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall

Fundamental Trig Identities

Reciprocal & Quotient

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Even/Odd

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

Pythagorean

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

TOPIC: INTRODUCTION TO TRIG IDENTITIES

PRACTICE

Simplify the expression.

(A)

$$\tan^2 \theta - \sec^2 \theta + 1$$

(B)

$$\frac{\tan(-\theta)}{\sec(-\theta)}$$

(C)

$$\left(\frac{\tan^2 \theta}{\sin^2 \theta} - 1 \right) \csc^2 \theta \cos^2(-\theta)$$

HOW TO: Fully Simplify Trig Expressions

Trig expressions are fully simplified when...

- ☐ all arguments are positive
- ☐ expression contains NO fractions
- ☐ expression contains as few trig fcn's as possible

Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall

Fundamental Trig Identities

Reciprocal & Quotient

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \end{aligned}$$

Even/Odd

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

Pythagorean

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

TOPIC: INTRODUCTION TO TRIG IDENTITIES

Verifying Trig Equations as Identities

◆ To verify that an equation is true, simplify ONE or BOTH sides with the goal of making them _____.

- ▶ ALWAYS start with the more _____ side first!

EXAMPLE

Verify the identity.

(A)

$$\frac{\sin \theta \cos \theta}{1 - \cos^2 \theta} = \frac{1}{\tan \theta}$$

(B)

$$\frac{\sec^2 \theta - \tan^2 \theta}{\cos(-\theta) + 1} = \frac{1 - \cos \theta}{\sin^2 \theta}$$

STRATEGIES: Simplifying Trig Expressions

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall

Fundamental Trig Identities

Reciprocal & Quotient

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Even/Odd

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

Pythagorean

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

TOPIC: INTRODUCTION TO TRIG IDENTITIES**EXAMPLE**

Verify the identity by working with one side.

$$\frac{1 - \sin \theta}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} = 0$$

STRATEGIES: Simplifying Trig Expressions

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall**Fundamental Trig Identities**Reciprocal & Quotient

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \end{aligned}$$

Even/Odd

$$\begin{aligned} \sin(-\theta) &= -\sin \theta \\ \cos(-\theta) &= \cos \theta \\ \tan(-\theta) &= -\tan \theta \end{aligned}$$

Pythagorean

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta \end{aligned}$$

**TOPIC: INTRODUCTION TO TRIG IDENTITIES****EXAMPLE**

Verify the identity by working with both sides.

$$\sec \theta (1 - \sin^2 \theta) = \frac{(1 + \tan^2 \theta) \cot^2 \theta}{\csc^2 \theta \sec \theta}$$

STRATEGIES: Simplifying Trig Expressions

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall**Fundamental Trig Identities**Reciprocal & Quotient

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \end{aligned}$$

Even/Odd

$$\begin{aligned} \sin(-\theta) &= -\sin \theta \\ \cos(-\theta) &= \cos \theta \\ \tan(-\theta) &= -\tan \theta \end{aligned}$$

Pythagorean

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta \end{aligned}$$

**TOPIC: INTRODUCTION TO TRIG IDENTITIES****PRACTICE**

Identify the most helpful first step in verifying the identity.

(A)

$$\left(\frac{\tan^2 \theta}{\sin^2 \theta} - 1 \right) = \sec^2 \theta \sin^2(-\theta)$$

STRATEGIES: Simplifying Trig Expressions

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall**Fundamental Trig Identities**Reciprocal & Quotient

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} & \sec \theta &= \frac{1}{\cos \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} \end{aligned}$$

Even/OddPythagorean

$$\begin{aligned} \sin(-\theta) &= -\sin \theta & \sin^2 \theta + \cos^2 \theta &= 1 \\ \cos(-\theta) &= \cos \theta & \tan^2 \theta + 1 &= \sec^2 \theta \\ \tan(-\theta) &= -\tan \theta & 1 + \cot^2 \theta &= \csc^2 \theta \end{aligned}$$

(B)

$$\sec^3 \theta = \sec \theta + \frac{\tan^2 \theta}{\cos \theta}$$

**TOPIC: SUM AND DIFFERENCE IDENTITIES****Sum and Difference of Sine & Cosine**

- ◆ The sum & difference identities are useful when you have multiple angles in the argument of a trig function.

TRIG IDENTITIES			
Name	Identity	Example	Use when...
Sum & Difference	$\sin(a + b) = \sin a \cos b + \cos a \sin b$	$\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$ OR $\sin(90^\circ + 30^\circ)$	argument contains a $\frac{\pi}{6}$ / 30° OR multiples of 15° or $\frac{\pi}{12}$
	$\sin(a - b) = \sin a \cos b - \cos a \sin b$		
	$\cos(a + b) = \cos a \cos b - \sin a \sin b$		
	$\cos(a - b) = \cos a \cos b + \sin a \sin b$		

- ◆ To find exact values of trig functions NOT on the unit circle, rewrite argument as sum or diff. of 2 KNOWN angles.

EXAMPLE

Find the exact value of the function.

$$\cos 15^\circ$$

**TOPIC: SUM AND DIFFERENCE IDENTITIES****EXAMPLE**

Rewrite each argument as the sum or difference of two angles on the unit circle.

(A) $\sin(75^\circ)$

(B) $\cos(-15^\circ)$

(C) $\cos\left(\frac{7\pi}{12}\right)$

PRACTICE

Find the exact value of the expression.

(A) $\cos 105^\circ$

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

(Sum/Difference Identities)

(B) $\sin 15^\circ$

(C) $\cos \frac{5\pi}{12}$

TOPIC: SUM AND DIFFERENCE IDENTITIES**EXAMPLE**

Find the exact value of the expression.

$$\sin 10^\circ \cos 20^\circ + \sin 20^\circ \cos 10^\circ$$

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

*(Sum/Difference Identities)***PRACTICE**

Find the exact value of the expression.

$$\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ$$

TOPIC: SUM AND DIFFERENCE IDENTITIES**EXAMPLE**

Expand the expression using the sum & difference identities and simplify.

(A) $\sin(\theta + 30^\circ)$

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

(Sum/Difference Identities)

(B) $\cos\left(\frac{\pi}{4} - \theta\right)$

PRACTICE

Expand the expression using the sum & difference identities and simplify.

$$\sin\left(-\theta - \frac{\pi}{2}\right)$$

**TOPIC: SUM AND DIFFERENCE IDENTITIES****Sum and Difference of Tangent**

- ◆ The sum & diff. identities for tangent can also be used to simplify expressions & find exact values of functions.

TRIG IDENTITIES			
Name	Identity	Example	Use when...
Sum & Diff.	$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$	$\tan\left(\pi + \frac{\pi}{4}\right)$	argument contains a + / - OR multiples of 15° or $\frac{\pi}{12}$
	$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$		

EXAMPLE

Expand the expression using the sum and/or difference identities, then simplify.

(A) $\tan(\theta - 45^\circ)$

(B) $\tan(90^\circ + \theta)$

- ◆ If $\tan a$ or $\tan b$ is undefined, rewrite $\tan(a \pm b)$ as $\frac{\sin(a \pm b)}{\cos(a \pm b)}$

TOPIC: SUM AND DIFFERENCE IDENTITIES**PRACTICE**

Find the exact value of the expression.

$$\tan 105^\circ$$

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

*(Sum/Difference Identities)***PRACTICE**

Expand the expression using the sum & difference identities and simplify.

$$\tan\left(-\theta - \frac{\pi}{2}\right)$$

**TOPIC: SUM AND DIFFERENCE IDENTITIES****Verifying Identities Using Sum and Difference Formulas**

◆ Recall: To verify an identity, simplify one or both sides to make them equal. Start with more complicated side.

EXAMPLE

Verify the identity using the sum/diff. identities.

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

STRATEGIES: Simplifying Trig Expressions

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(Sum/Difference Identities)

EXAMPLE

Verify the identity using the sum/difference identities.

$$\cot(90^\circ - \theta) = \tan \theta$$

TOPIC: SUM AND DIFFERENCE IDENTITIES**EXAMPLE**

Verify the identity.

(A)

$$\frac{\cos(a+b)}{\cos a \cos b} = -\tan a \tan b + 1$$

STRATEGIES: Simplifying Trig Expressions

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(Sum/Difference Identities)

(B)

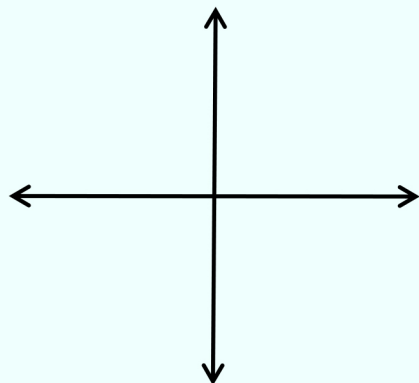
$$\frac{\sin(a-b)}{\tan a \tan b} = \cos a \cos b (\cot b - \cot a)$$

**TOPIC: SUM AND DIFFERENCE IDENTITIES****Evaluate Sums and Differences Given Conditions**

◆ Sometimes you'll need to evaluate a sum or difference based on given trig values without knowing the angles.

EXAMPLE

Find $\sin(a + b)$ given $\cos a = \frac{4}{5}$, $\sin b = \frac{5}{13}$,
where a is in Quadrant IV & b is in Quadrant II.

**HOW TO: Evaluate Trig Functions Given Conditions**

- 1) _____ identity & **identify** unknown trig value(s)
- 2) From given info, sketch & label* $\angle(s)$ in *proper quadrant*
- 3) Find _____(s) using Pythag. Thm.*
- 4) **Solve** for unknown trig value(s) from (1)
- 5) Plug in values & simplify

*pay attention to _____

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

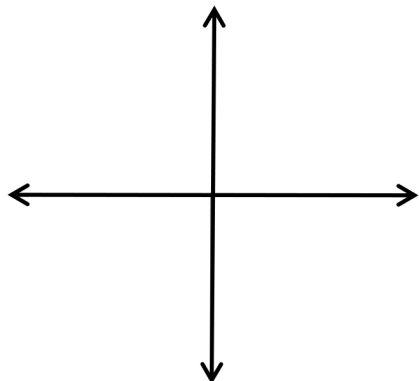
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(Sum/Difference Identities)

TOPIC: SUM AND DIFFERENCE IDENTITIES**EXAMPLE**

Find $\sin(a + b)$ given $\sin a = \frac{3}{5}$, $\sin b = -\frac{1}{2}$, where $\frac{\pi}{2} < a < \pi$ & $\frac{3\pi}{2} < b < 2\pi$

**HOW TO: Evaluate Trig Functions Given Conditions**

- 1) Expand identity & **identify** unknown trig value(s)
- 2) From given info, sketch & label* $\angle(s)$ in *proper quadrant*
- 3) Find missing sides using Pythag. Thm.*
- 4) **Solve** for unknown trig value(s) from (1)
- 5) Plug in values & simplify

**pay attention to SIGN*

Recall

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

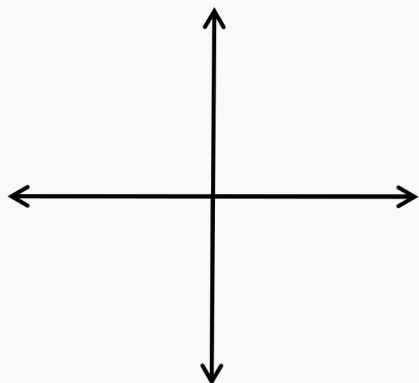
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

(Sum/Difference Identities)

PRACTICE

Find $\cos(a + b)$ given $\cos a = \frac{1}{2}$, $\sin b = \frac{1}{2}$, & a is in QIV & b is in QII.





TOPIC: DOUBLE ANGLE IDENTITIES

Double Angle Identities

◆ When the sum identities are used with the *same* two angles, we get the **double angle identities**.

TRIG IDENTITIES			
Name	Identity	Example	Use when...
Sum & Diff.	$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$ $\sin(\theta + \theta) = \sin \theta \cos \theta + \cos \theta \sin \theta$		
Double Angle	$\sin 2\theta = 2 \underline{\hspace{2cm}}$	$\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12} =$	argument contains $\underline{\hspace{2cm}}$ OR you recognize a $\underline{\hspace{2cm}}$ of the identity
	$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= 1 - 2 \sin^2 \theta$ $= 2 \cos^2 \theta - 1$		
	$\tan 2\theta = \frac{2 \tan \theta}{1 - \underline{\hspace{2cm}}^2 \theta}$		

◆ To simplify expressions & verify identities, you'll need to recognize different **forms** of the Double Angle Identities.

EXAMPLE

Simplify the expression, do not evaluate.

$$\sin 15^\circ \cos 15^\circ$$

TOPIC: DOUBLE ANGLE IDENTITIES**EXAMPLE**

Use the double angle formulas to verify the identity.

(A)

$$\frac{\sin 2\theta}{\tan \theta} = \cos 2\theta + 1$$

STRATEGIES: Simplifying Trig Expressions

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

Recall

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

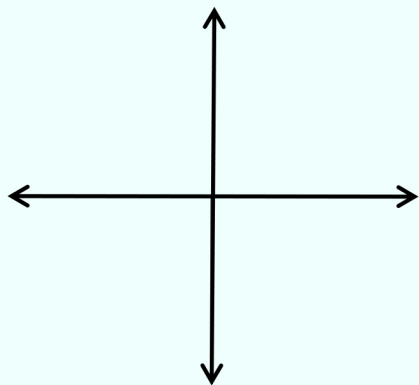
$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

*(Double Angle Identities)***(B)**

$$\cot 2\theta = \cot \theta - \frac{1}{2 \cos \theta \sin \theta}$$

TOPIC: DOUBLE ANGLE IDENTITIES**EXAMPLE**

Given $\cos \theta = \frac{4}{5}$ and $0 < \theta < \frac{\pi}{2}$, find $\sin(2\theta)$.

**HOW TO: Evaluate Trig Functions Given Conditions**

- 1) Expand identity & **identify** unknown trig value(s)
- 2) From given info, sketch & label* $\triangle(s)$ in *proper quadrant*
- 3) Find missing sides using Pythag. Thm.*
- 4) **Solve** for unknown trig value(s) from (1)
- 5) Plug in values & simplify

*pay attention to _____

Recall

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

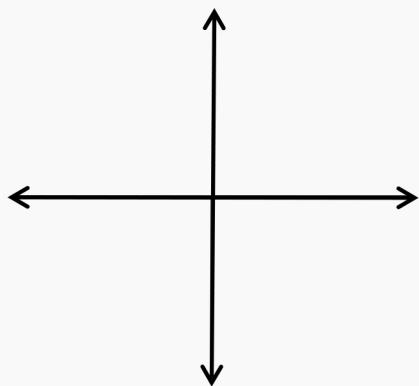
$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

(Double Angle Identities)

PRACTICE

Given $\tan \theta = \frac{5}{12}$ and $0 < \theta < \frac{\pi}{2}$, find $\cos(2\theta)$.





TOPIC: LINEAR TRIGONOMETRIC EQUATIONS

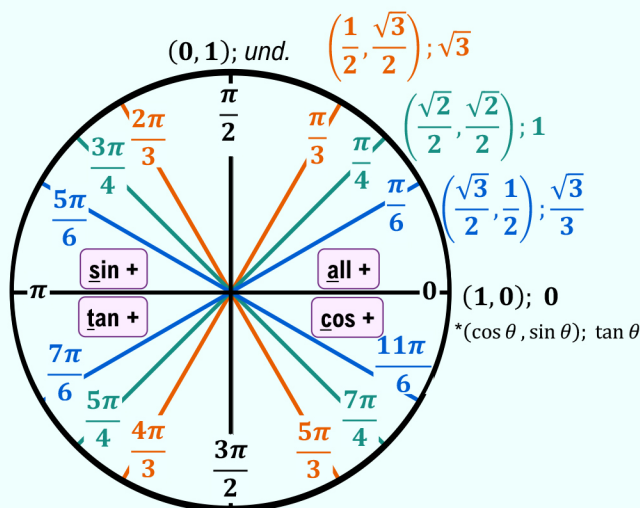
Introduction to Trig Equations

◆ Trig equations have _____ solutions.

EXAMPLE

Find all solutions to the equation within the interval $[0, 2\pi]$.

$$\sin \theta = \frac{1}{2}$$



◆ To find *all* sol'ns to a trig eqn, first find all sol'ns on the unit circle, then add _____ to each, where n is an integer.

EXAMPLE

Find all solutions to the equation.

(A)

$$\sin \theta = \frac{1}{2}$$

(B)

$$\cos x = -1$$

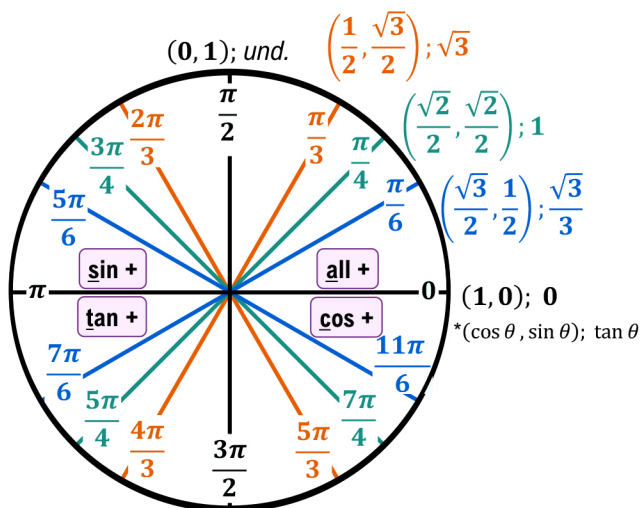
EXAMPLE

Find all solutions to the equation.

$$\tan \theta = \sqrt{3}$$



TOPIC: LINEAR TRIGONOMETRIC EQUATIONS



PRACTICE

Find all solutions to the equation.

(A) $\cos x = 1$

(B) $\tan \theta = 1$

(C) $\sin \theta = -\frac{\sqrt{3}}{2}$

EXAMPLE

Find all solutions to the equation in degrees.

$\sin \theta = \frac{1}{2}$

Recall

$$\theta_d = \frac{180^\circ}{\pi} \cdot \theta_r$$

TOPIC: LINEAR TRIGONOMETRIC EQUATIONS

How to Solve Linear Trigonometric Equations

◆ To solve linear trig equations, isolate the trig function, just like a _____.

Recall	Linear Equations	New	Linear TRIG Equations
	$4x - 3 = 1$		$4 \sin \theta - 3 = 1$
	$4x = 4$		
	$x = 1$		$\sin \theta = 1$

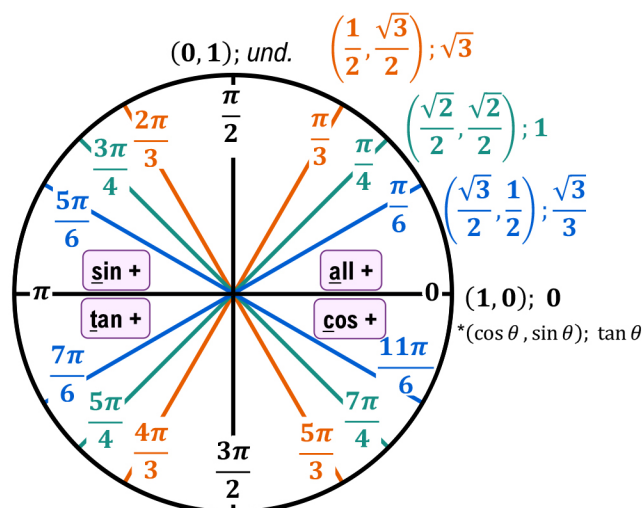
◆ Remember, trig equations have **multiple** solutions.

EXAMPLE Find all solutions to the equation within $[0, 2\pi]$.

$$-2 \cos \theta + \sqrt{3} = 0$$

HOW TO: Solve Linear Trig Eqns

- 1) Isolate trig function using $+$ $-$ $\times$ $\div$
- 2) Find all solutions on the unit circle
- 3) If domain is *not* restricted: Add $2\pi n$ to each solution
- 4) Isolate θ

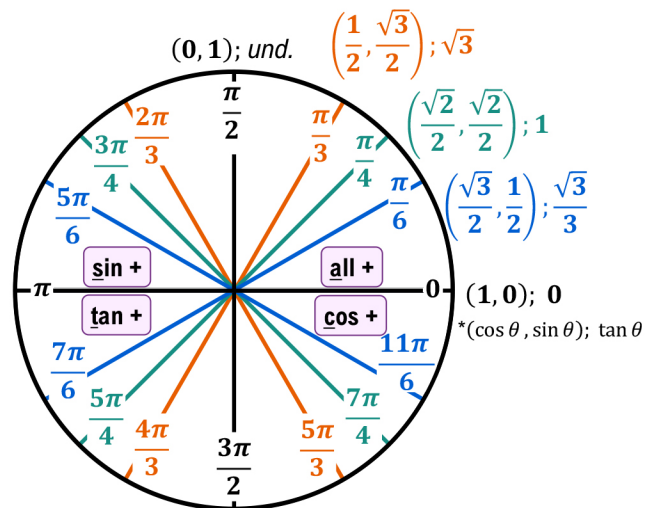




TOPIC: LINEAR TRIGONOMETRIC EQUATIONS

HOW TO: Solve Linear Trig Eqns

- 1) Isolate trig function using $+$ $-$ $\times$ $\div$
- 2) Find all solutions on the unit circle
- 3) If domain is *not* restricted: Add $2\pi n$ to each solution
- 4) Isolate θ



PRACTICE

Find all solutions to the equation.

(A) $3 \sin \theta - 6 = -9$

(B) $\sqrt{2} \cdot \cos \theta + 4 = 5$

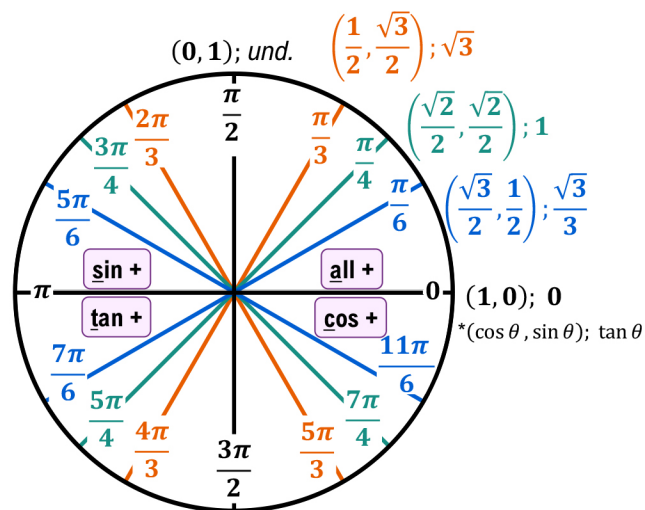
(C) $\sqrt{3} \cdot \tan \theta - 7 = -6$



TOPIC: LINEAR TRIGONOMETRIC EQUATIONS

HOW TO: Solve Linear Trig Eqns

- 1) Isolate trig function using $+$ $-$ $\times$ $\div$
- 2) Find all solutions on the unit circle
- 3) If domain is *not* restricted: Add $2\pi n$ to each solution
- 4) Isolate θ



EXAMPLE

Find all solutions to the equation.

$$4 \cos(2\theta + \pi) + 8 = 12$$



TOPIC: SOLVING TRIG EQUATIONS USING IDENTITIES

Solve Trig Equations Using Identity Substitutions

◆ Recall: Solve trig equations by finding θ that makes the equation true.

- When given eqns with *multiple* trig fcn, use _____ to rewrite in terms of *one* trig fcn, then solve.

Recall	Linear Trig Equations	New	Other Trig Equations
	$\tan \theta = 1$ $\downarrow$ $\theta = \frac{\pi}{4} + \pi n$		$\frac{\sec^2 \theta - 1}{\tan \theta} = 1$ $\downarrow$ $\frac{\quad}{\tan \theta} = 1$ $\downarrow$ $\tan \theta = 1$ $\downarrow$ $\theta = \frac{\pi}{4} + \pi n$ Substitute using _____ Identity

EXAMPLE

Find all solutions to the equation.

$$\frac{\sin 2\theta}{\cos(-\theta)} = 1$$

Rewrite [TOP | BOTTOM] using _____ Identity

Rewrite [TOP | BOTTOM] using _____ Identity

Solve _____ trig equation

**TOPIC: SOLVING TRIG EQUATIONS USING IDENTITIES****PRACTICE**

Find all solutions to the equation.

$$(\cos \theta + \sin \theta)(\cos \theta - \sin \theta) = -\frac{1}{2}$$

PRACTICEFind all solutions to the equation where $0 \leq \theta \leq 2\pi$.

$$\sin \theta \cos(2\theta) - \sin(2\theta) \cos \theta = \frac{\sqrt{2}}{2}$$



CHAPTER RESOURCE: THE UNIT CIRCLE

