



Study Prep

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TOPIC: INTRODUCTION TO ANGLES

Angles in Standard Position

- ◆ **Angles:** The “gap” between 2 line segments/sides, measured in degrees ($^{\circ}$) from ____ to ____ on the x/y plane.
- ▶ Always draw *from* [INITIAL | TERMINAL] side (____-axis) to [INITIAL | TERMINAL] side (“standard position”).
 - ▶ Draw **positive** angles in the [CCW ↺ | CW ↻] direction, **negative** angles in the [CCW ↺ | CW ↻] direction.

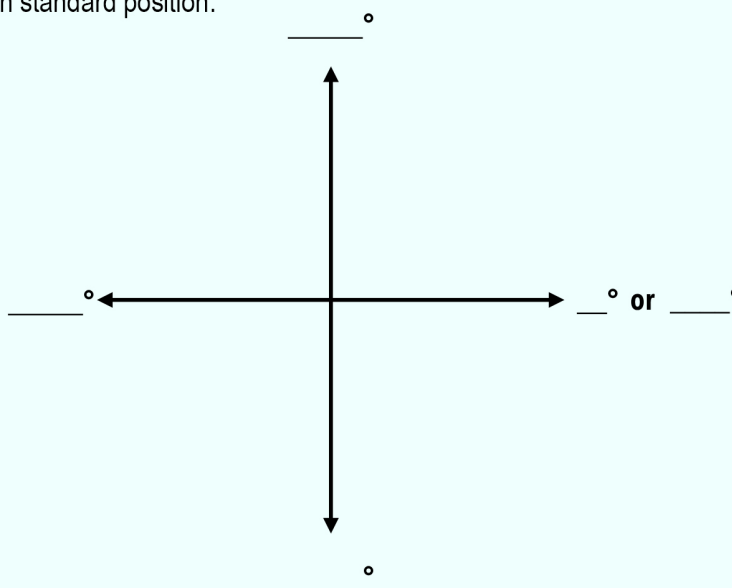
EXAMPLE

Sketch each angle in standard position.

(A) 60°

(B) 150°

(C) -60°

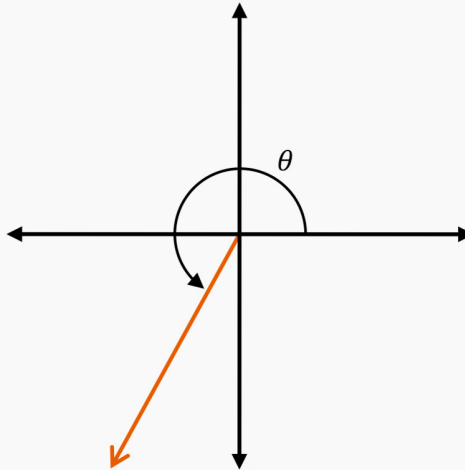


Types of Angles		
Acute ($< 90^{\circ}$) 	Obtuse ($> 90^{\circ}$) 	Right ($= 90^{\circ}$)

TOPIC: INTRODUCTION TO ANGLES**PRACTICE**

What is the approximate measure of the angle shown below? Choose the most reasonable answer.

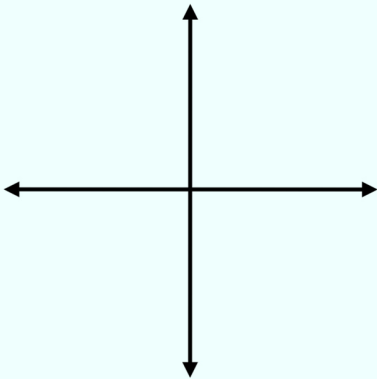
- (A) 60°
(B) 150°
(C) 240°
(D) 300°

**EXAMPLE**

Sketch each angle in standard position.

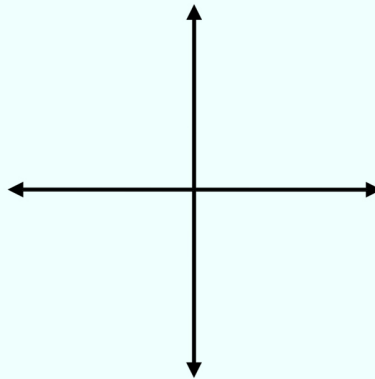
(A)

45°



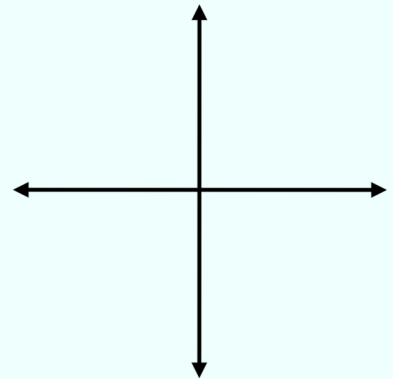
(B)

210°



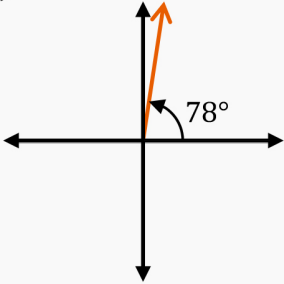
(C)

-100°

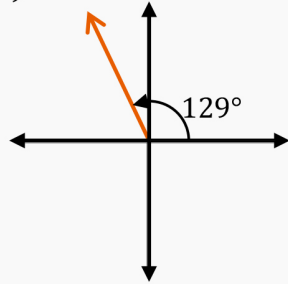


TOPIC: INTRODUCTION TO ANGLES**PRACTICE**Which angle is **NOT** a positive angle drawn in standard position?

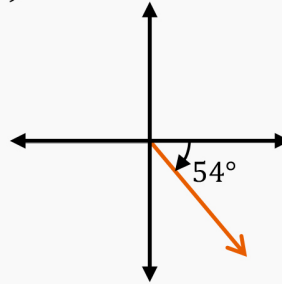
(A)



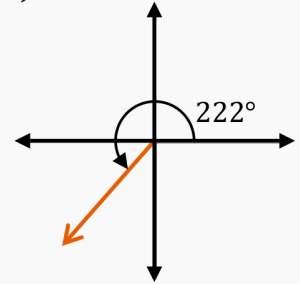
(B)



(C)



(D)



TOPIC: COTERMINAL ANGLES**Introduction to Coterminal Angles**

◆ You'll sometimes need to draw angles which are NOT between 0° & 360° .

- ▶ Angles are **COTERMINAL** if they point in the same direction, i.e. same [**INITIAL** | **TERMINAL**] side.
- ▶ To find angles which are coterminal with a given angle, add or subtract multiples of _____.

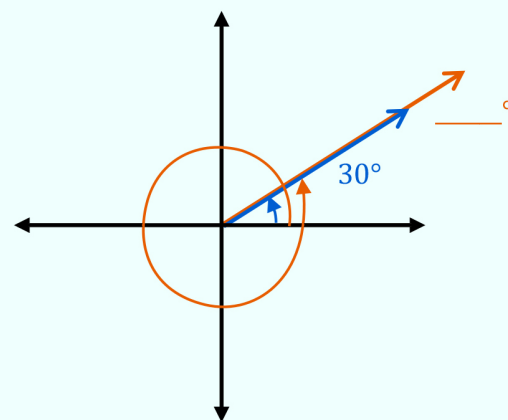
EXAMPLE

Find an angle between 0° & 360° which is coterminal with the given angle.

(A) 390°

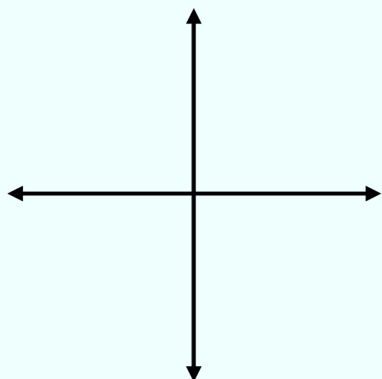
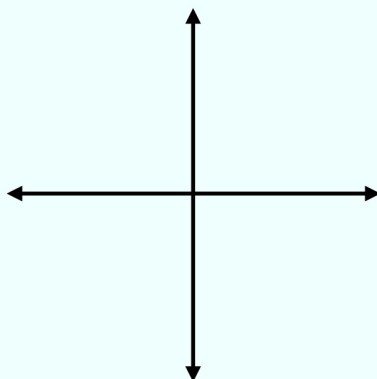
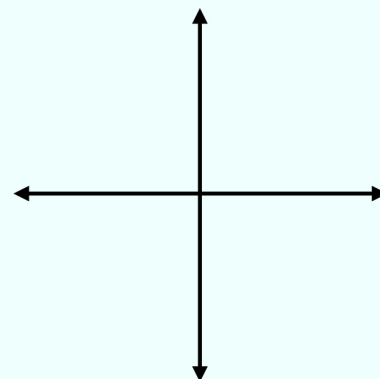
(B) -270°

(C) 1000°



**TOPIC: COTERMINAL ANGLES****EXAMPLE**

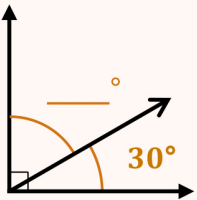
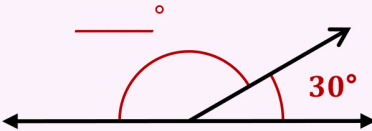
Find the smallest positive angle coterminal with the given angle. Sketch the angle in standard position.

(A) 710° **(B)** -37° **(C)** -480° 

TOPIC: COMPLEMENTARY & SUPPLEMENTARY ANGLES

Introduction to Complementary & Supplementary Angles

◆ **Complementary** angles are two angles that add to _____, while **Supplementary** angles add to _____.

Complementary Angles	Supplementary Angles
 $\theta_1 + \theta_2 = \text{_____}^\circ$ Complementary angles form right Corners.	 $\theta_1 + \theta_2 = \text{_____}^\circ$ Supplementary angles form Straight lines.

EXAMPLE

Find the complement & the supplement of the given angle, if possible.

(A)

20°

Complement: _____

Supplement: _____

(B)

100°

Complement: _____

Supplement: _____

Note: Complementary & supplementary angles are always assumed to be *positive*.

PRACTICE

Find the complement & supplement of a 45° angle.

Complement: _____

Supplement: _____

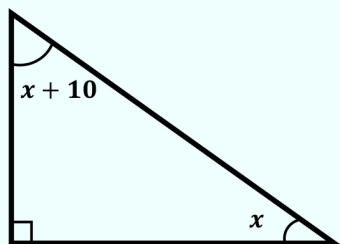
Ch. 5 - Trigonometric Functions

TOPIC: COMPLEMENTARY & SUPPLEMENTARY ANGLES**Solving Problems with Complementary & Supplementary Angles**

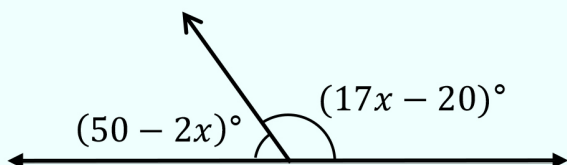
- ◆ When problems have angles written in terms of variables (like x), use complementary & supplementary angles to solve.
 - ▶ In a right triangle, because one angle is always 90° , the other two angles are always _____.

EXAMPLE

Find each of the angles in the triangle below.

**EXAMPLE**

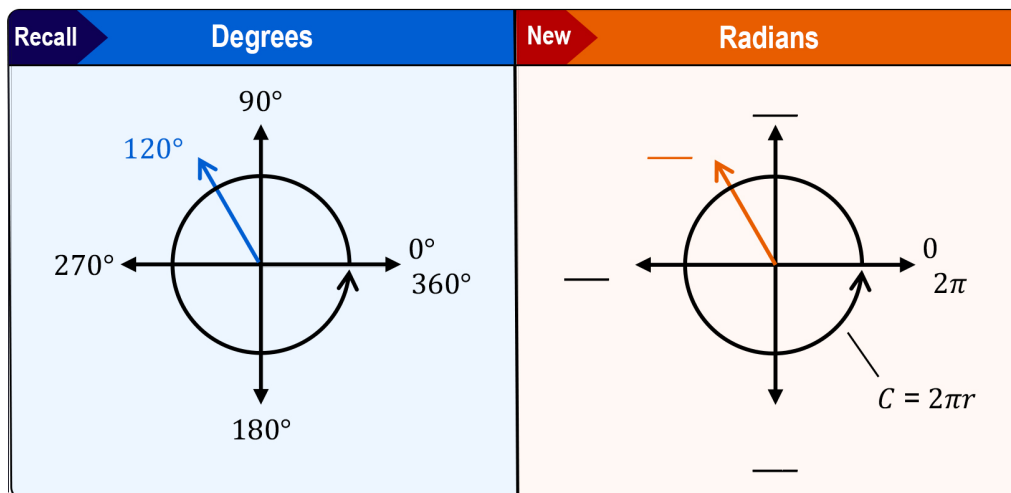
Solve for each angle below.



TOPIC: RADIANS

Converting between Degrees & Radians

◆ **Radians:** A different unit for measuring angles, based on a circle's circumference. Full circle = $360^\circ = \underline{\hspace{1cm}}$ radians



$$\begin{aligned}
 &\text{New} \quad \theta_{rad} = \underline{\hspace{1cm}} \cdot \theta_{deg} \quad \text{(degrees to radians)} \\
 &\text{New} \quad \theta_{deg} = \underline{\hspace{1cm}} \cdot \theta_{rad} \quad \text{(radians to degrees)}
 \end{aligned}$$

EXAMPLE

Convert the angle from degrees to radians or from radians to degrees.

(A)

$$120^\circ$$

(B)

$$\frac{6\pi}{5}$$



Ch. 5 - Trigonometric Functions

TOPIC: RADIANS**PRACTICE**Convert the angle 540° from degrees to radians.**Recall**

$$\theta_r = \frac{\pi}{180^\circ} \cdot \theta_d$$

(degrees to radians)

$$\theta_d = \frac{180^\circ}{\pi} \cdot \theta_r$$

(radians to degrees)

PRACTICEConvert the angle $-\frac{5\pi}{6}$ from radians to degrees.**Recall**

$$\theta_r = \frac{\pi}{180^\circ} \cdot \theta_d$$

(degrees to radians)

$$\theta_d = \frac{180^\circ}{\pi} \cdot \theta_r$$

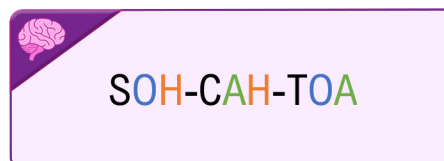
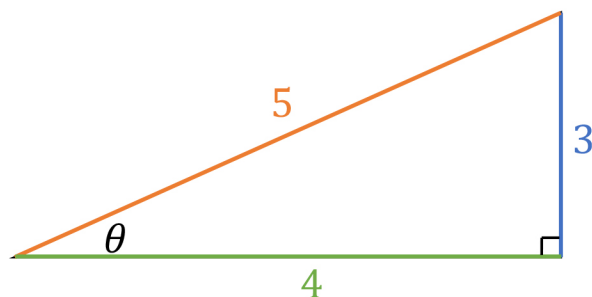
(radians to degrees)



TOPIC: RIGHT TRIANGLE TRIGONOMETRY

Introduction to Trigonometric Functions

- ◆ **Trig Functions** relate _____ to side lengths in right triangles.
- ▶ The three main trig functions are Sine, Cosine, & Tangent which are _____.



New Trig Functions

SOH

$$\sin \theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\sin \theta = \text{---}$$

CAH

$$\cos \theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

$$\cos \theta = \text{---}$$

TOA

$$\tan \theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \text{---}$$

EXAMPLE

Find the value of the trig function indicated, given the triangle.

(A)

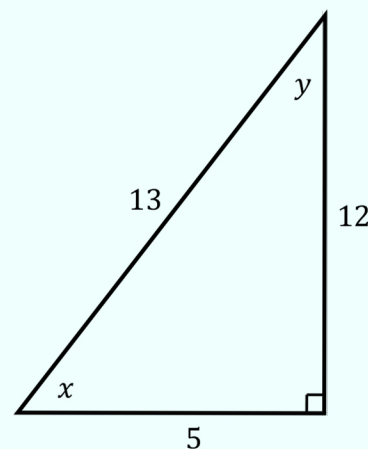
$$\sin x$$

(B)

$$\tan x$$

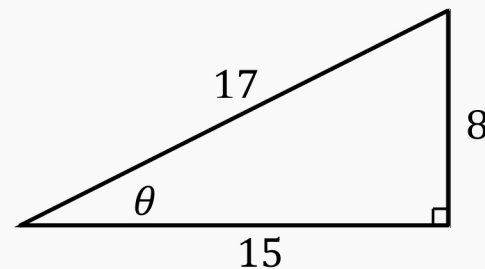
(C)

$$\cos y$$

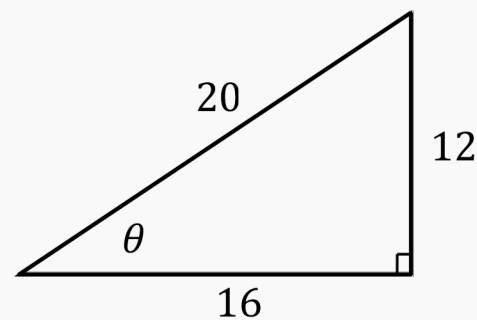


TOPIC: RIGHT TRIANGLE TRIGONOMETRY**PRACTICE**

Given the right triangle below, evaluate $\cos(\theta)$.

**PRACTICE**

Given the right triangle below, evaluate $\tan(\theta)$.



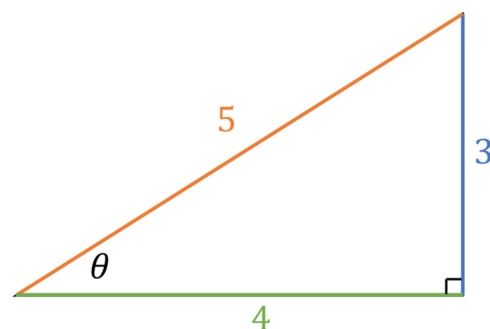
TOPIC: RIGHT TRIANGLE TRIGONOMETRY

Fundamental Trigonometric Identities

◆ The other three trig functions are cosecant, secant, & cotangent.

▶ These are _____ of the other trig functions.

Recall	Trig Functions	New	Reciprocal Identities
	$\sin \theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{3}{5}$		$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{Hyp}}{\text{Opp}} = \frac{5}{3}$
	$\cos \theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{4}{5}$		$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hyp}}{\text{Adj}} = \frac{5}{4}$
	$\tan \theta = \frac{\text{Opp}}{\text{Adj}} = \frac{3}{4}$		$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{Adj}}{\text{Opp}} = \frac{4}{3}$
	$\tan \theta = \frac{\sin \theta}{\cos \theta}$		$\cot \theta = \frac{\cos \theta}{\sin \theta}$



EXAMPLE

Find the value of the trig function indicated, given the triangle.

(A)

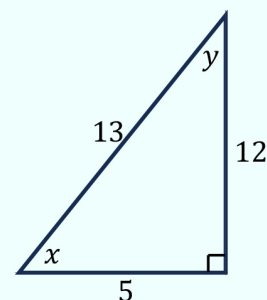
$\sec x$

(B)

$\csc x$

(C)

$\cot y$



**TOPIC: RIGHT TRIANGLE TRIGONOMETRY****PRACTICE**

If $\tan \theta = \frac{12}{5}$, find the values of the five other trigonometric functions. Rationalize the denominators if necessary.

PRACTICE

If $\sin \theta = \frac{\sqrt{17}}{17}$, find the values of the five other trigonometric functions. Rationalize the denominators if necessary.

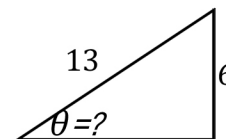


TOPIC: RIGHT TRIANGLE TRIGONOMETRY

Introduction to Inverse Trigonometric Functions

◆ For some problems, you will be *given* the value of trig function, & have to *find* the angle.

- ▶ You'll need the _____ of the trig function to find the angle.

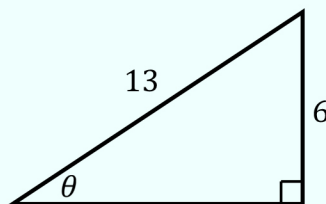


Recall	Exponents	New	Trig Functions
	$ \begin{array}{c} x^2 \\ \text{Operation} \\ 3 \xrightarrow{\quad} 3^2 = 9 \\ \text{Inverse} \\ \sqrt{x^2} \end{array} $ The inverse _____ the operation.		$ \begin{array}{c} \sin(\theta) \\ \text{Operation} \\ 30^\circ \xrightarrow{\quad} \sin(30) = \frac{1}{2} \\ \text{Inverse} \\ \sin^{-1}(\sin(\theta)) \end{array} $ The inverse trig function _____ the trig operation.

◆ Inverse trig functions are also called “_____” functions, so $\sin^{-1}(\theta)$ can also be written as “_____”

EXAMPLE

Write a trigonometric function to represent the angle θ .

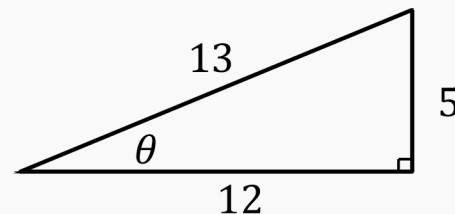


HOW TO: USING INVERSE TRIG TO FUNCTIONS FIND ANGLES

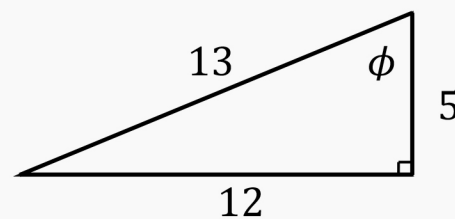
- 1) Choose a trig function which includes the correct _____ and sides
- 2) Write equation with the chosen trig function
- 3) Take the inverse on _____ sides to isolate the angle

TOPIC: RIGHT TRIANGLE TRIGONOMETRY**PRACTICE**

Given the right triangle below, use the sine function to write a trigonometric expression for the missing angle θ .

**PRACTICE**

Given the right triangle below, use the cosine function to write a trigonometric expression for the missing angle ϕ .



**TOPIC: RIGHT TRIANGLE TRIGONOMETRY****How to Use a Calculator for Trig Functions**

◆ For certain problems you will need to use a calculator to evaluate the function, rather than using fractions.

- ▶ For trig functions, use the **sin** **cos** and **tan** buttons on the calculator.
- ▶ Make sure your calculator is in the correct **MODE** (_____ or radian) when solving problems.

EXAMPLE

Find the value for each of the following trigonometric operations and round to the nearest tenth.

(A)

$$\sin(37^\circ)$$

[Radian | Degree]

(B)

$$\tan\left(\frac{2\pi}{15}\right)$$

[Radian | Degree]

(C)

$$\sec(50^\circ)$$

[Radian | Degree]

(D)

$$\arctan\left(\frac{3}{4}\right) * \text{degrees}$$

[Radian | Degree]

◆ For _____ trig functions press the **2nd** button, then the associated trig function.

**TOPIC: RIGHT TRIANGLE TRIGONOMETRY****PRACTICE**

What is the positive value of A in the interval $[0^\circ, 90^\circ)$ that will make the following statement true?

Express the answer in four decimal places.

$$\sin A = 0.9235$$

PRACTICE

What is the positive value of P in the interval $[0^\circ, 90^\circ)$ that will make the following statement true?

Express the answer in four decimal places.

$$\cot P = 5.2371$$

PRACTICE

What is the positive value of D in the interval $[0, \frac{\pi}{2})$ that will make the following statement true?

Express the answer in four decimal places.

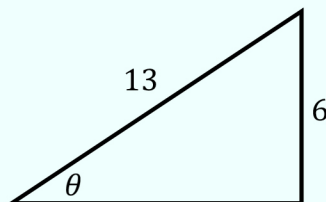
$$\sec D = 3.2842$$

TOPIC: RIGHT TRIANGLE TRIGONOMETRY**EXAMPLE**

Determine the missing angle θ in degrees for the right triangle below (approximate your answer to 2 decimal places).

$$\theta = \sin^{-1}\left(\frac{6}{13}\right)$$

[RADIAN | DEGREE]

**HOW TO: Using Inverse Trig Functions to Find Angles with Calculator**

- 1) Choose a trig function which includes the correct **angles** and sides
- 2) Write equation with the chosen trig function
- 3) Take the inverse on **both** sides to isolate the angle
- 4) Press the **2nd** key, and the associated trig function to get the inverse trig function.
- 5) Approximate the inverse trig function using a calculator



TOPIC: SPECIAL RIGHT TRIANGLES

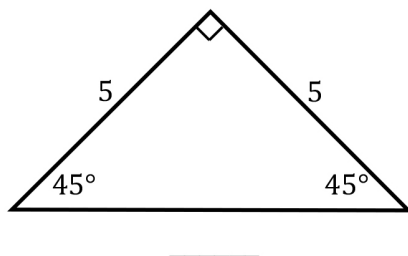
45-45-90 Triangles

◆ In triangles with 45° angles, the 2 legs are always the _____ length.

- ▶ The hypotenuse will always be a multiple of the leg length, which you can find using:

Recall

$$a^2 + b^2 = c^2$$



New

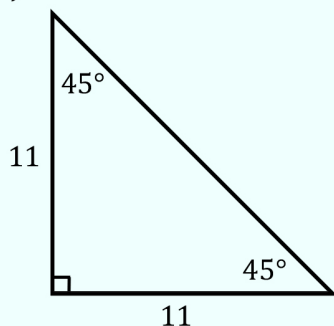
$$\text{hyp} = \text{leg} \cdot \underline{\hspace{1cm}}$$

(45 - 45 - 90)

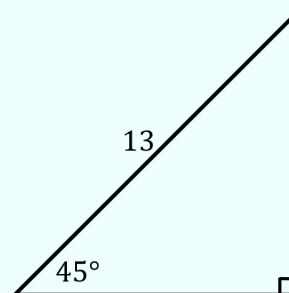
EXAMPLE

Solve for the unknown side(s) of each triangle.

(A)



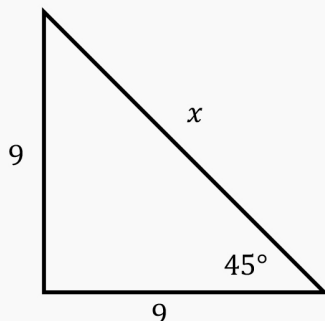
(B)



Ch. 5 - Trigonometric Functions

TOPIC: SPECIAL RIGHT TRIANGLES**PRACTICE**

Given the triangle below, determine the missing side(s) without using the Pythagorean theorem (make sure your answer is fully simplified).

**PRACTICE**

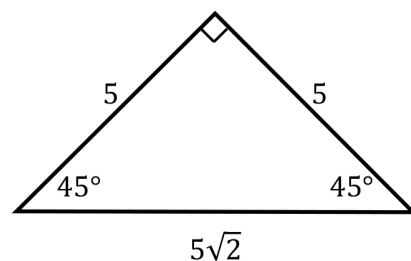
Without using a calculator, determine all values of P in the interval $[0, 90^\circ)$ with the following trigonometric function value.

$$\csc P = \sqrt{2}$$

TOPIC: SPECIAL RIGHT TRIANGLES

Common Trig Functions For 45-45-90 Triangles

- ◆ The common trig functions follow a specific pattern for 45-45-90 triangles.

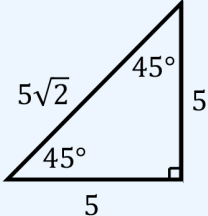
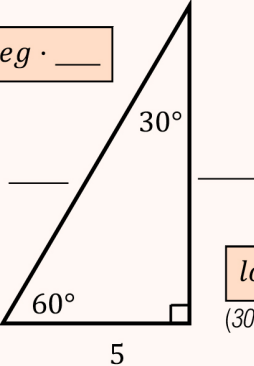


New Trig Function Values for 45-45-90 Triangle					
sin	$= \frac{\text{Opp}}{\text{Hyp}} =$		csc	$= \frac{1}{\sin(\theta)} =$	
cos	$= \frac{\text{Adj}}{\text{Hyp}} =$		sec	$= \frac{1}{\cos(\theta)} =$	
tan	$= \frac{\text{Opp}}{\text{Adj}} =$		cot	$= \frac{1}{\tan(\theta)} =$	

TOPIC: SPECIAL RIGHT TRIANGLES

30-60-90 Triangles

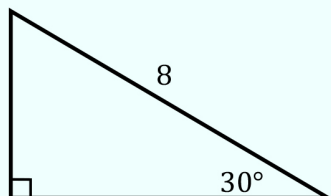
◆ For the 30-60-90 triangle, relate side lengths to the *shortest* leg.

Recall	45 – 45 – 90	New	30 – 60 – 90
	 $hyp = leg \cdot \sqrt{2}$ (45 – 45 – 90)		 $hyp = short\ leg \cdot \underline{\hspace{1cm}}$ (30 – 60 – 90) $long\ leg = short\ leg \cdot \underline{\hspace{1cm}}$ (30 – 60 – 90)

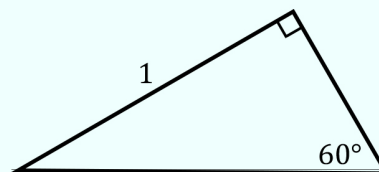
EXAMPLE

Solve for the unknown sides of each triangle.

(A)



(B)

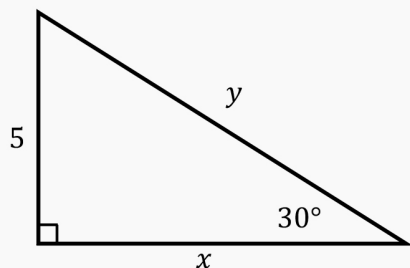


Ch. 5 - Trigonometric Functions

TOPIC: SPECIAL RIGHT TRIANGLES

PRACTICE

Given the triangle below, determine the missing side(s) without using the Pythagorean theorem (make sure your answer is fully simplified).



PRACTICE

Without using a calculator, determine all values of A in the interval $\left[0, \frac{\pi}{2}\right)$ with the following trigonometric function value.

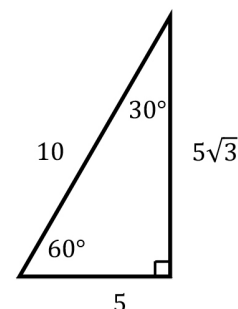
$$\cos A = \frac{\sqrt{3}}{2}$$

TOPIC: SPECIAL RIGHT TRIANGLES

Common Trig Functions For 30-60-90 Triangles

- ◆ The common trig functions follow a specific pattern for 30-60-90 triangles.

New Trig Function Values for 30-60-90 Triangle			
sin	30° $= \frac{\text{Opp}}{\text{Hyp}} =$		60°
cos	30° $= \frac{\text{Adj}}{\text{Hyp}} =$		60°
tan	30° $= \frac{\text{Opp}}{\text{Adj}} =$		60°
csc	30° $= \frac{1}{\sin(\theta)} =$		60°
sec	30° $= \frac{1}{\cos(\theta)} =$		60°
cot	30° $= \frac{1}{\tan(\theta)} =$		60°





TOPIC: COFUNCTIONS OF COMPLEMENTARY ANGLES

Cofunction Identities

◆ Angles which add to 90° are **Complementary**.

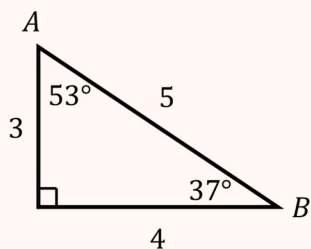
- In a right triangle, the _____ angles are *ALWAYS* complementary.

Complementary Angle Theorem

Cofunctions of complementary angles are equal.

New

Cofunction Identities



$$\sin A = \cos B \quad \sec A = \csc B \quad \tan A = \cot B$$

$$*B = 90^\circ - A \text{ or } \frac{\pi}{2} - A$$

$$\sin 37^\circ = \frac{3}{5} = \cos \underline{\hspace{2cm}}$$

EXAMPLE

Write the expression in terms of the appropriate cofunction.

(A)

$$\sin(37^\circ)$$

[Radian | Degree]

(B)

$$\tan\left(\frac{2\pi}{15}\right)$$

[Radian | Degree]

(C)

$$\sec(50^\circ)$$

[Radian | Degree]

(D)

$$\arctan\left(\frac{3}{4}\right) * \text{degrees}$$

[Radian | Degree]

**TOPIC: COFUNCTIONS OF COMPLEMENTARY ANGLES****PRACTICE**

Write the expression in terms of the appropriate cofunction.

$$\cos\left(\frac{19\pi}{45}\right)$$

PRACTICE

Write the expression in terms of the appropriate cofunction.

$$\cot(25^\circ)$$

TOPIC: COFUNCTIONS OF COMPLEMENTARY ANGLESSolving Equations Using Cofunction Identities

◆ To solve trig equations, rewrite one side, then set arguments of the functions equal.

$$\sin(x - 10) = \cos(x)$$

$$\sin(\underbrace{x - 10}) = \sin(\underbrace{90 - x})$$

$$x - 10 = 90 - x$$

Recall

Cofunction Identities

$$\sin A = \cos(90^\circ - A)$$

$$\sec A = \csc(90^\circ - A)$$

$$\tan A = \cot(90^\circ - A)$$

EXAMPLE

Solve for θ in the following equation.

$$\cos(\theta) = \sin(2\theta - 30^\circ)$$

HOW TO: Solve Trig Equations Using Cofunctions

- 1) Use cofunction identities to get the _____ trig functions on both sides of the = sign
- 2) Set the _____ of the functions equal
- 3) Solve for the missing variable.

**TOPIC: COFUNCTIONS OF COMPLEMENTARY ANGLES****PRACTICE**

Find the acute angle solution to the following equation involving cofunctions. θ is in degrees.

$$\cos(2\theta + 15) = \sin(5\theta + 12)$$

PRACTICE

Find the acute angle solution to the following equation involving cofunctions. P is in degrees.

$$\sec\left(\frac{4P}{5} + 20\right) = \csc\left(\frac{5P}{8} + \frac{223}{4}\right)$$

PRACTICE

Find the acute angle solution to the following equation involving cofunctions. M is in degrees.

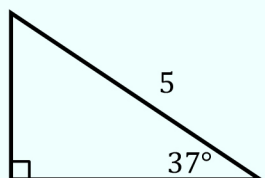
$$\tan\left(\frac{M}{2} + 5\right) = \cot(M - 5)$$

**TOPIC: SOLVING RIGHT TRIANGLES****Finding Missing Side Lengths**

◆ If given one _____ & one _____, find missing side lengths using trig equations & Pythagorean Theorem.

EXAMPLE

Find all side lengths of the given triangle.

**HOW TO SOLVE: RIGHT TRIANGLES**
(GIVEN 1 SIDE & 1 ANGLE)

1) Find other angle using $\angle B = \text{---} - \angle A$

2) Choose a trig function so

$$\text{any trig}(\angle A) = \frac{\text{given side}}{x} \text{ or}$$

$$\text{any trig}(\angle A) = \frac{x}{\text{given side}}$$

3) Solve equation for x (2nd side length)

4) Use Pythagorean Theorem to find 3rd side

Recall

$$a^2 + b^2 = c^2$$



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TOPIC: COFUNCTIONS OF COMPLEMENTARY ANGLES

PRACTICE

A right triangle with an angle of 31° has a hypotenuse of 10. Calculate the side of the triangle opposite to the 31° angle (y), and the side adjacent to the 31° angle (x). Round your answer to 3 decimal places.

**TOPIC: SOLVING RIGHT TRIANGLES****EXAMPLE**

The Grand Lighthouse on a coastal cliff stands 288 m tall and is positioned approximately 2.3 km inland from the shore of the Sea. A seafarer on a sailboat directly in front of the lighthouse observes the top of the structure and records the angle of elevation as 3.4° . Determine the distance (in kilometers) of the sailboat from the coastline to two decimal places.

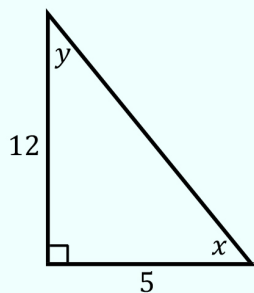
**TOPIC: SOLVING RIGHT TRIANGLES****Finding Missing Angles**

◆ If you're given 2+ _____ lengths of a triangle, find the missing angle(s) using trig & inverse trig equations.

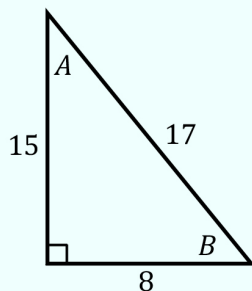
EXAMPLE

Find all angles of the given triangle.

(A)



(B)

**HOW TO SOLVE RIGHT TRIANGLES**

(GIVEN 2+ SIDES)

- 1) If missing a side, use Pythagorean theorem
- 2) For ANY non-90° angle, write the trig equation with the known sides
- 3) Take the inverse trig fn of each side of eqn & solve for $\angle$
- 4) Find other angle using $\angle B = 90^\circ - \angle A$

Recall

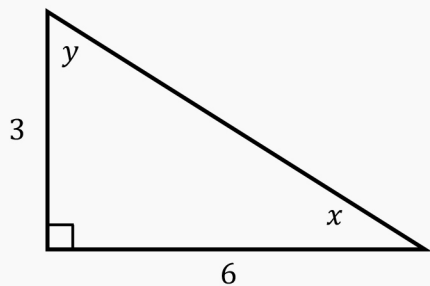
$$a^2 + b^2 = c^2$$



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**TOPIC: SOLVING RIGHT TRIANGLES****PRACTICE**

Given the right triangle below, calculate all missing angles in degrees (round your answer to 3 decimal places).





TOPIC: SOLVING RIGHT TRIANGLES

EXAMPLE

A hiking path can be traced from a mountain lodge (at an elevation of 6500 feet) to a scenic viewpoint in a canyon (at an elevation of 4300 feet). The path spans 4400 feet. Determine the angle of inclination of the hiking path.

TOPIC: DEFINING THE UNIT CIRCLE

Introduction to the Unit Circle

◆ **Unit Circle:** Circle of radius 1 relating **angles** from 0 to 360° (or ____ radians) to **x & y values**. Centered at (____ , ____).

Recall

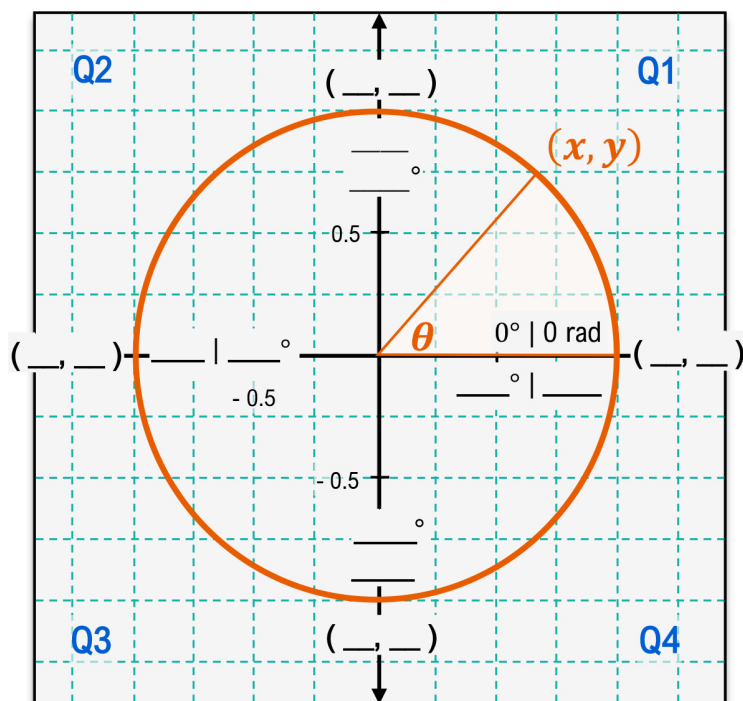
Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

New

Unit Circle

$$x^2 + y^2 = 1$$



EXAMPLE

Identify which points are on the unit circle and label them on the graph.

(A)

$(1, 1)$

[ON | NOT ON] unit circle

(B)

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

[ON | NOT ON] unit circle

PRACTICE

Identify the quadrant that the given angle is located in.

(A)

$\frac{7\pi}{4}$ radians

Quadrant: ____

(B)

$\frac{\pi}{7}$ radians

Quadrant: ____

(C)

$\frac{2\pi}{3}$ radians

Quadrant: ____

(D)

$\frac{6\pi}{5}$ radians

Quadrant: ____

TOPIC: DEFINING THE UNIT CIRCLE**PRACTICE**

Test whether the point is on the unit circle by plugging it into the equation.

$$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$$

New**Unit Circle**

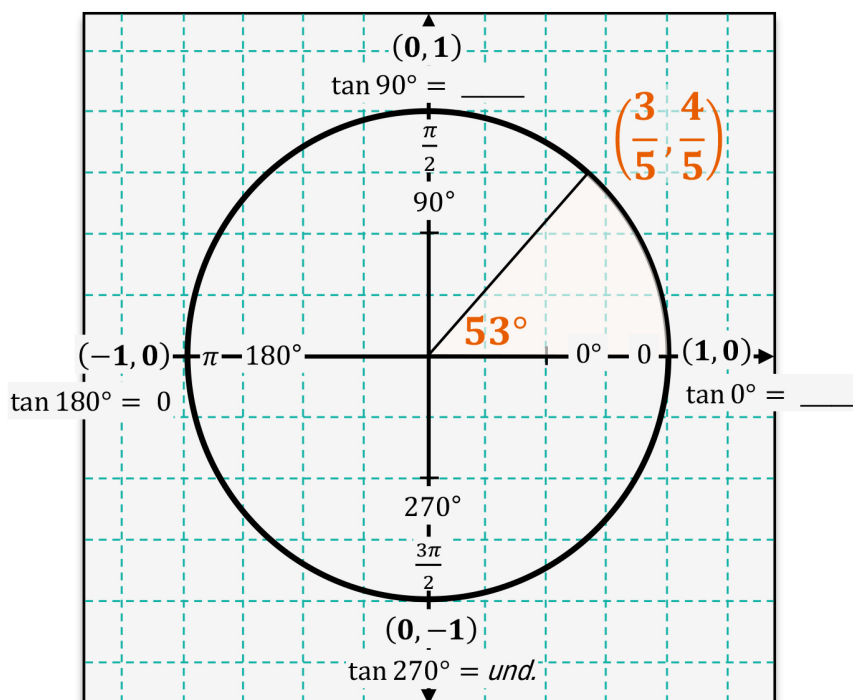
$$x^2 + y^2 = 1$$

TOPIC: TRIG FUNCTIONS ON THE UNIT CIRCLE

Sine, Cosine, & Tangent on the Unit Circle

◆ **Trigonometric Functions** relate angles to _____ on the unit circle. On the unit circle:

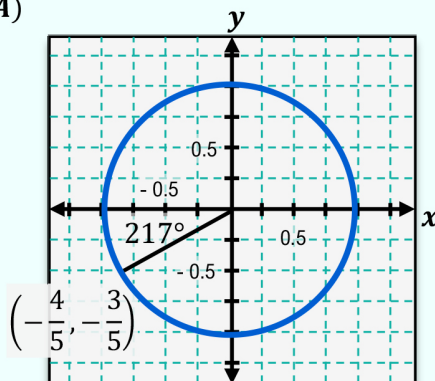
- ▶ The **SIN** of an angle is *ALWAYS* the _____ value or the _____ of the corresponding triangle.
- ▶ The **COS** of an angle is *ALWAYS* the _____ value or the _____ of the corresponding triangle.
- ▶ The **TAN** of an angle is *ALWAYS* _____



EXAMPLE

Find the sine, cosine, and tangent of each angle using the unit circle.

(A)

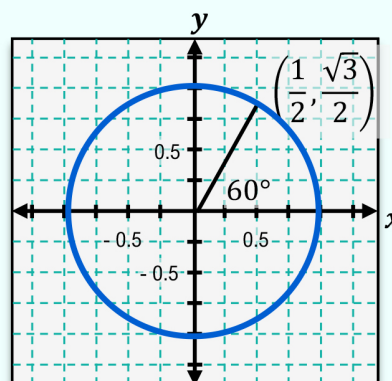


$$\sin 217^\circ = \underline{\hspace{2cm}}$$

$$\cos 217^\circ = \underline{\hspace{2cm}}$$

$$\tan 217^\circ = \underline{\hspace{2cm}}$$

(B)



$$\sin 60^\circ = \underline{\hspace{2cm}}$$

$$\cos 60^\circ = \underline{\hspace{2cm}}$$

$$\tan 60^\circ = \underline{\hspace{2cm}}$$

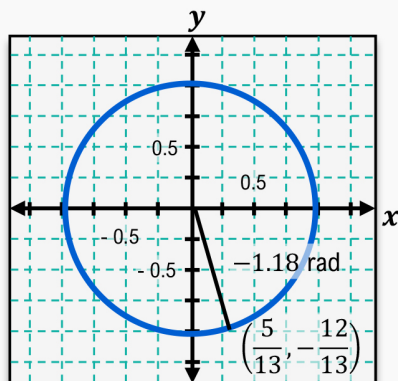


TOPIC: TRIG FUNCTIONS ON THE UNIT CIRCLE

PRACTICE

Find the sine, cosine, and tangent of each angle using the unit circle.

(A)

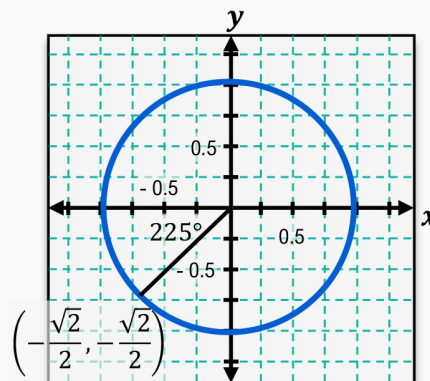


$$\sin \theta = \underline{\hspace{2cm}}$$

$$\cos \theta = \underline{\hspace{2cm}}$$

$$\tan \theta = \underline{\hspace{2cm}}$$

(B)



$$\sin \theta = \underline{\hspace{2cm}}$$

$$\cos \theta = \underline{\hspace{2cm}}$$

$$\tan \theta = \underline{\hspace{2cm}}$$

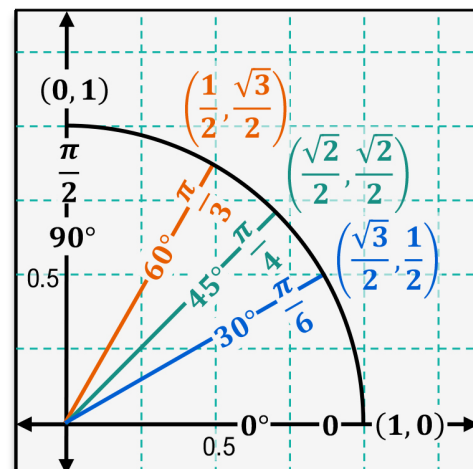
TOPIC: RECIPROCAL TRIG FUNCTIONS ON THE UNIT CIRCLE

Cosecant, Secant, & Cotangent on the Unit Circle

◆ Recall: Besides Sine, Cosine, & Tangent, there are 3 other trig functions: Cosecant, Secant, & Cotangent.

- These are the reciprocal trig functions, so find them by taking the reciprocal of sin, cos, & tan values on the unit circle.

Recall	SIN, COS, & TAN	New	SEC, CSC, & COT
	$\sin \theta = y$		$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{y}$
	$\cos \theta = x$		$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{x}$
	$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$		$\cot \theta = \frac{1}{\tan \theta} = \frac{x}{y}$



EXAMPLE

Evaluate each expression.

(A)

$$\csc \frac{\pi}{6}$$

(B)

$$\cot \frac{\pi}{4}$$

(C)

$$\sec 0$$

PRACTICE

Evaluate each expression.

(A)

$$\cot \frac{11\pi}{6}$$

(B)

$$\csc 225^\circ$$

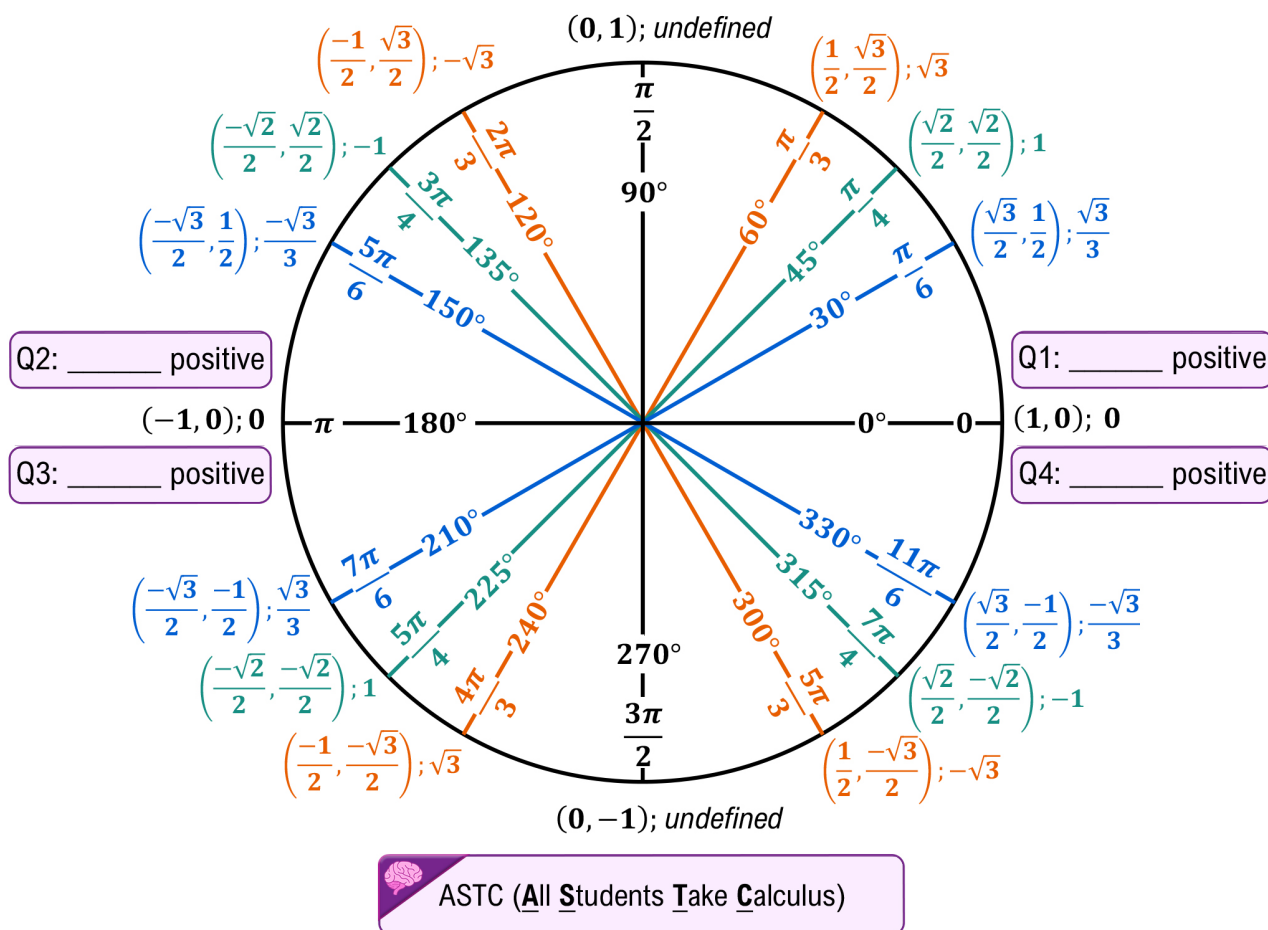
(C)

$$\sec \frac{\pi}{3}$$

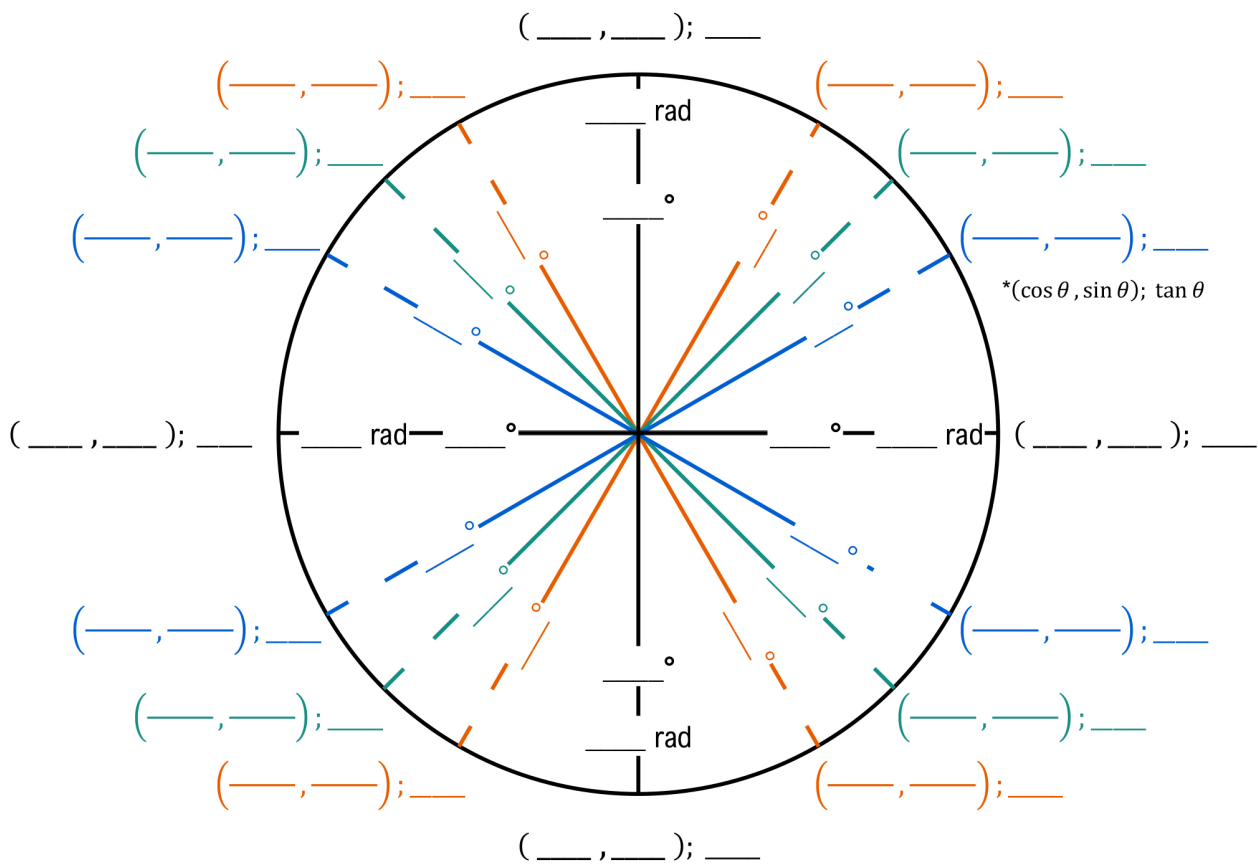


CHAPTER RESOURCE

The Unit Circle: Filled In



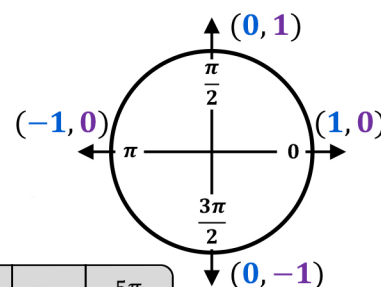
The Unit Circle: Blank



TOPIC: GRAPHS OF SINE & COSINE

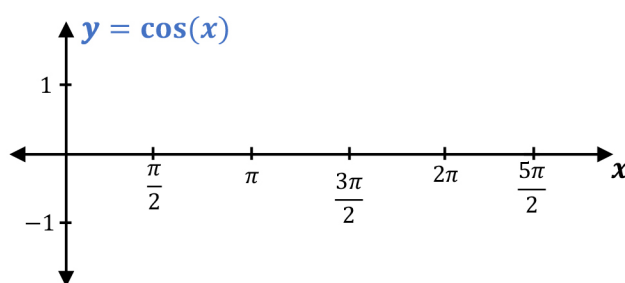
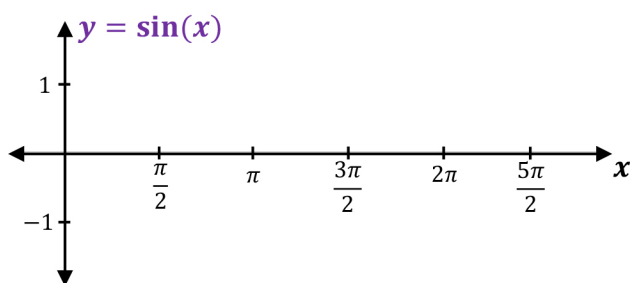
Graphing Sine & Cosine (with Vertical Shifts)

- ◆ **Sine** & **Cosine** values _____ around the unit circle, so their graphs are _____.
 ▶ The high points are “crests” or _____; the low points are “troughs” or _____.



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
$\sin x$						

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
$\cos x$						



- ◆ Just as we've vertically shifted functions, we can also vertically shift $\sin$ & $\cos$ by adding a constant _____.
 ▶ For positive k , graph shifts [UP | DOWN]; for negative k , graph shifts [UP | DOWN].

Recall Transformations

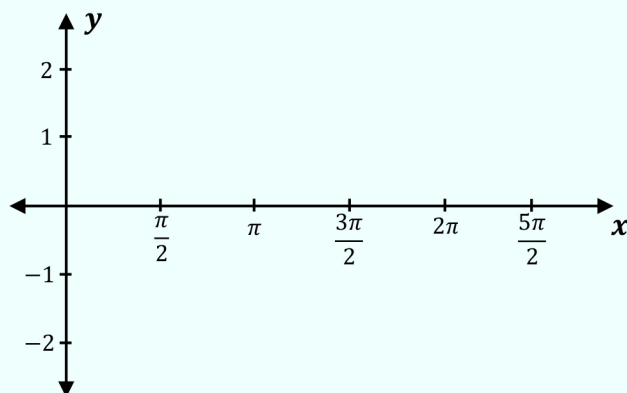
$$g(x) = f(x) + k$$

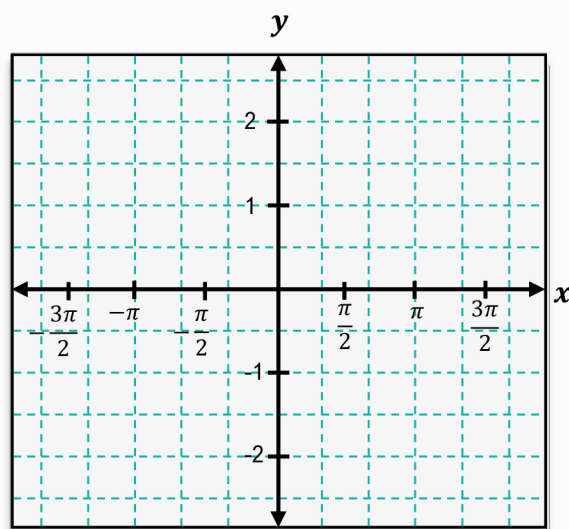
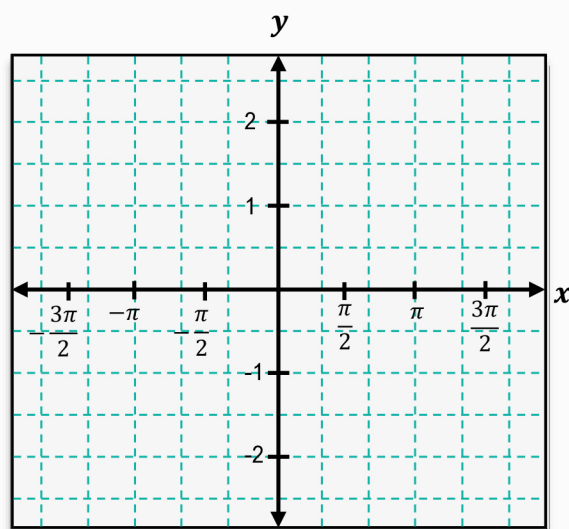
$$y = \sin(x) + \underline{\hspace{1cm}}$$

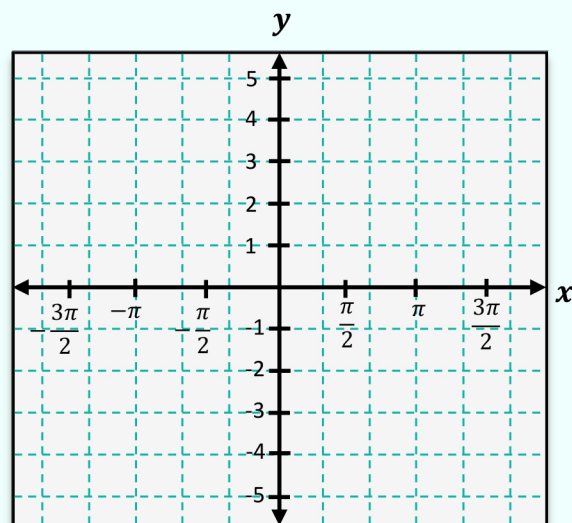
$$y = \cos(x) + \underline{\hspace{1cm}}$$

EXAMPLE

Graph the function $y = \sin(x) + 1$.



TOPIC: GRAPHS OF SINE & COSINE**PRACTICE**Sketch the function $y = \cos(x) - 1$ on the graph below.**PRACTICE**Determine the value of $y = \sin\left(-\frac{\pi}{2}\right) + 50$ without using a calculator or the unit circle.

TOPIC: GRAPHS OF SINE & COSINE**EXAMPLE**Graph the function $y = \sin(x) + 3$.

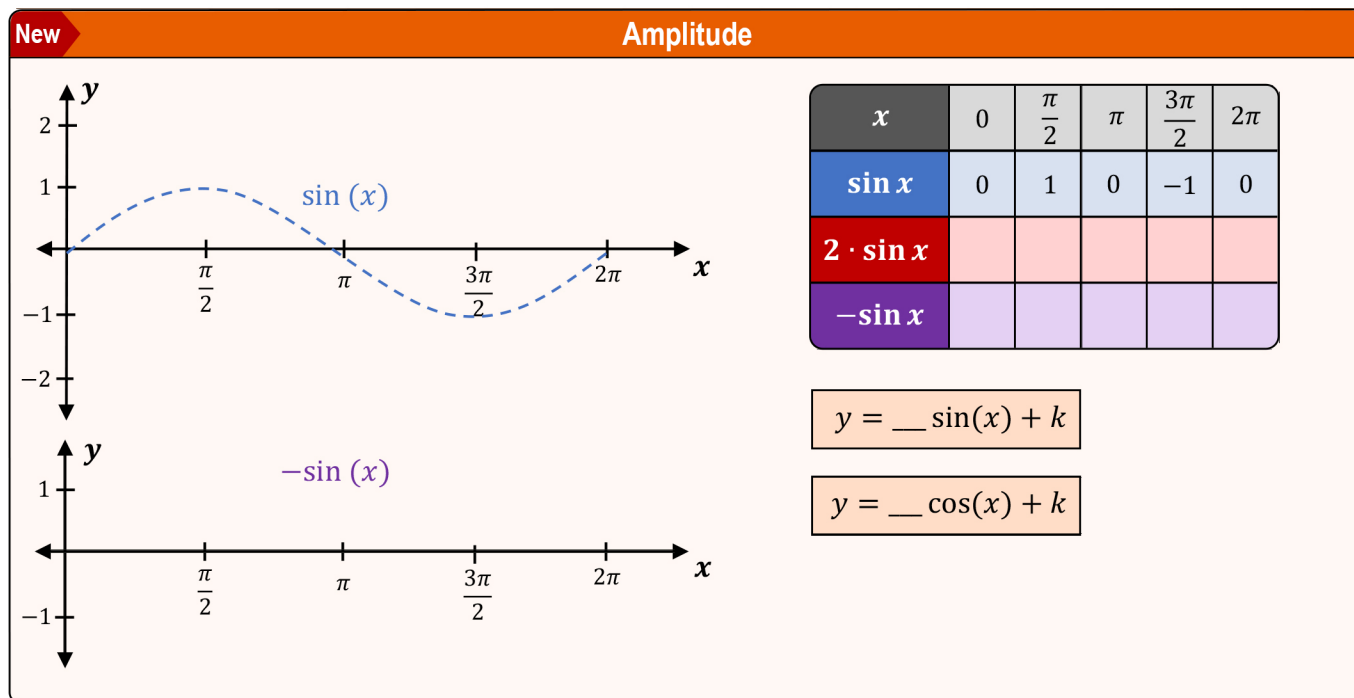


TOPIC: GRAPHS OF SINE & COSINE

Amplitude & Reflection of Sine & Cosine

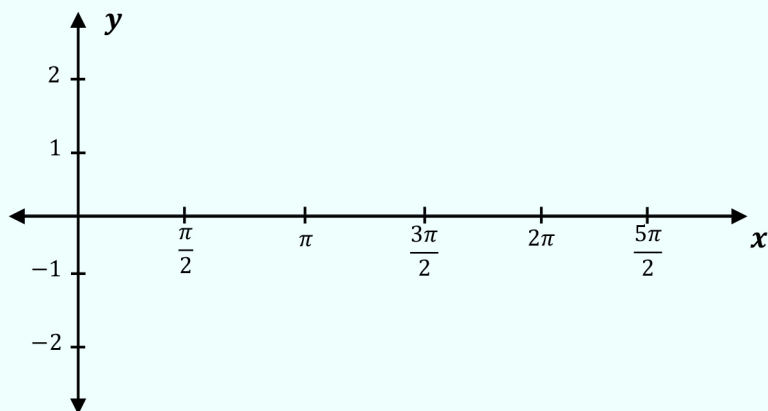
◆ Recall: Sine & Cosine graphs are repeating waves.

- ▶ **Amplitude:** # affecting how _____ the peaks are, i.e. the distance from the midline to the peaks or valleys.
- ▶ If the amplitude is negative, the graph is _____ over the x-axis.



EXAMPLE

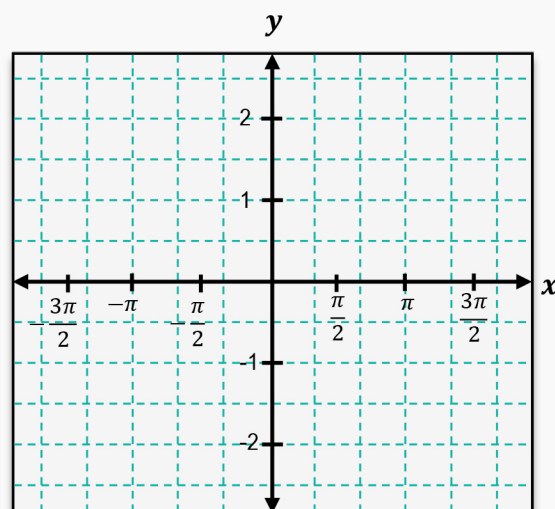
Graph the function $y = -\frac{3}{2} \cdot \cos x$



TOPIC: GRAPHS OF SINE & COSINE

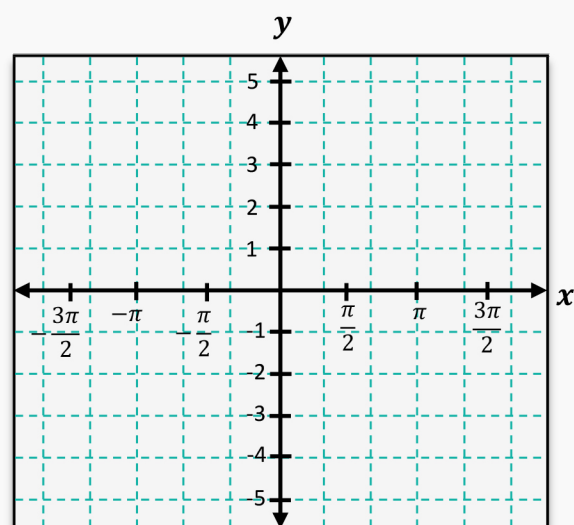
PRACTICE

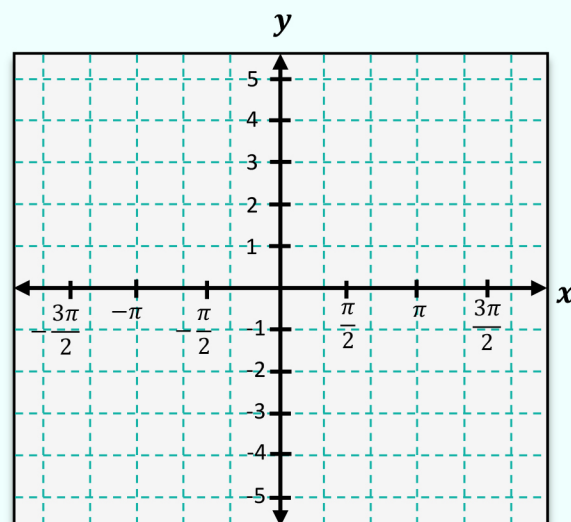
Determine the value of $y = -2 \cdot \sin\left(-\frac{3\pi}{2}\right) + 10$ without using a calculator or the unit circle.



PRACTICE

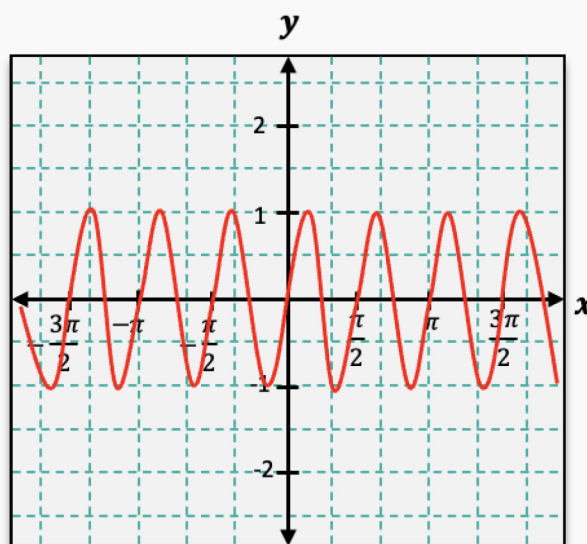
Graph the function $y = -3 \cdot \cos(x)$.



TOPIC: GRAPHS OF SINE & COSINE**EXAMPLE**Graph the function $y = 2 \cdot \sin(x) - 1$.

TOPIC: GRAPHS OF SINE & COSINE**PRACTICE**

Given below is the graph of the function $y = \sin(bx)$. Determine the correct value for b .

**PRACTICE**

The period for the function $y = \cos(bx)$ is $T = 20\pi$. Determine the correct value for b .

TOPIC: PHASE SHIFTS OF SINE & COSINE

Phase Shifts

◆ **Phase shift (h):** a _____ shift (left/right), indicated by numbers **[INSIDE | OUTSIDE]** parentheses.

- ▶ A phase shift can make a *cosine* graph look like a _____ graph.

New
Phase Shifts of Sine & Cosine Graphs

$\cos(x)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	1	0	-1	0	1
$\cos\left(x - \frac{\pi}{2}\right)$					

$y = A \sin (bx - \text{---}^*) + k$

$y = A \cos (bx - \text{---}^*) + k$

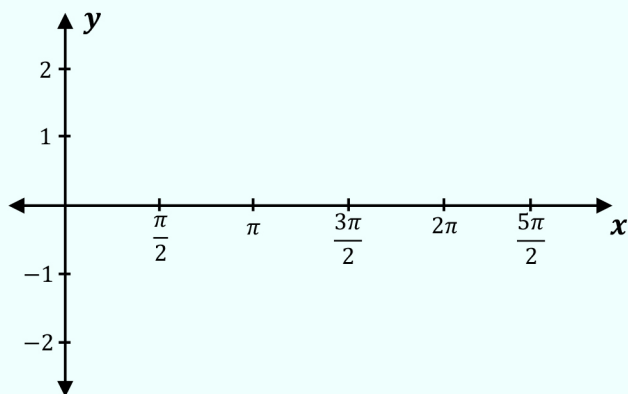
*you may see other letters for this (ϕ, C, d)

If given $(bx - \#)$ h is positive and graph shifts $\left[\leftarrow \mid \rightarrow\right]$ by $\frac{h}{b}$

If given $(bx + \#)$ h is negative and graph shifts $\left[\leftarrow \mid \rightarrow\right]$ by $\frac{h}{b}$

EXAMPLE

Graph $y = \sin (2x + \pi)$ over one full period.





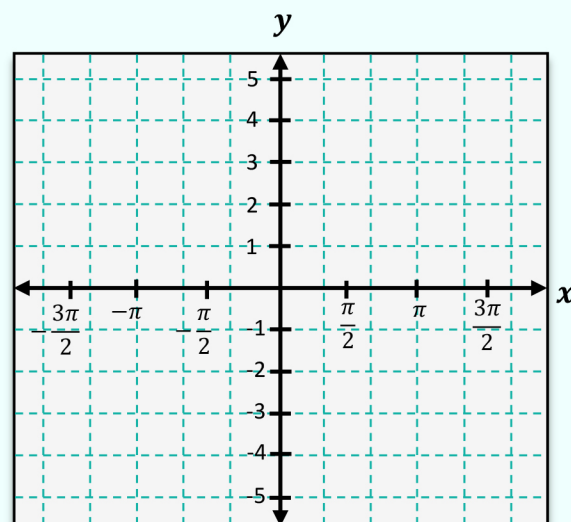
TOPIC: PHASE SHIFTS OF SINE & COSINE

PRACTICE

Describe the phase shift for the following function: $y = \cos \left(5x - \frac{\pi}{2} \right)$

PRACTICE

Describe the phase shift for the following function: $y = \cos \left(2x + \frac{\pi}{6} \right)$

TOPIC: PHASE SHIFTS OF SINE & COSINE**EXAMPLE**Graph the function $y = 3 \cdot \sin(x + \pi)$.



TOPIC: GRAPHS OF SECANT & COSECANT

Graphing the Secant & Cosecant Functions

◆ Because they are reciprocals, we can use $\sin$ & $\cos$ to graph $\csc$ & $\sec$.

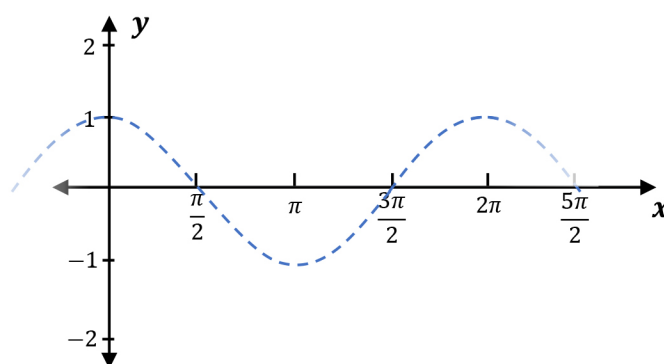
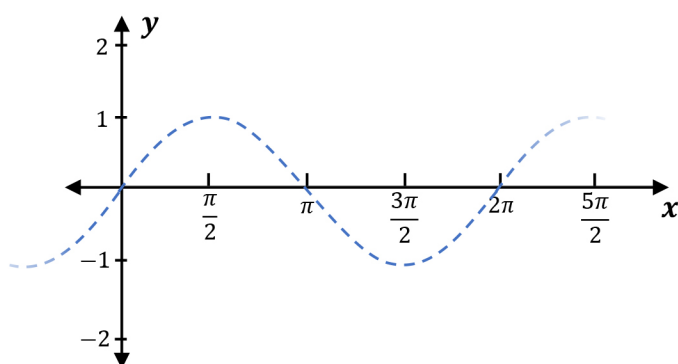
▶ When $\sin$ & $\cos = 0$, $\csc$ & $\sec$ are undefined, so they have _____ and approach ∞ .

Recall Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x}$$

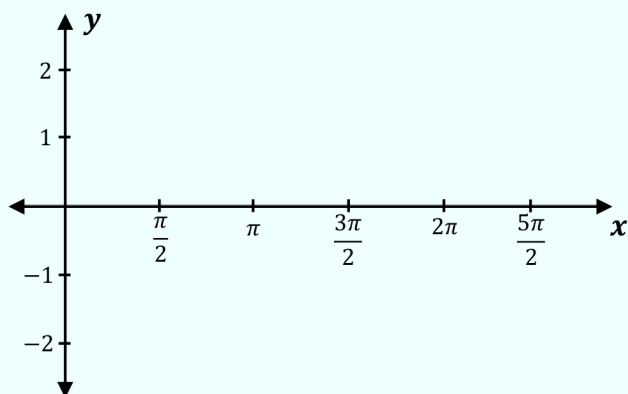
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
$\sin x$	0	1	0	-1	0	1
$\csc x$						

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
$\cos x$	1	0	-1	0	1	0
$\sec x$						



EXAMPLE

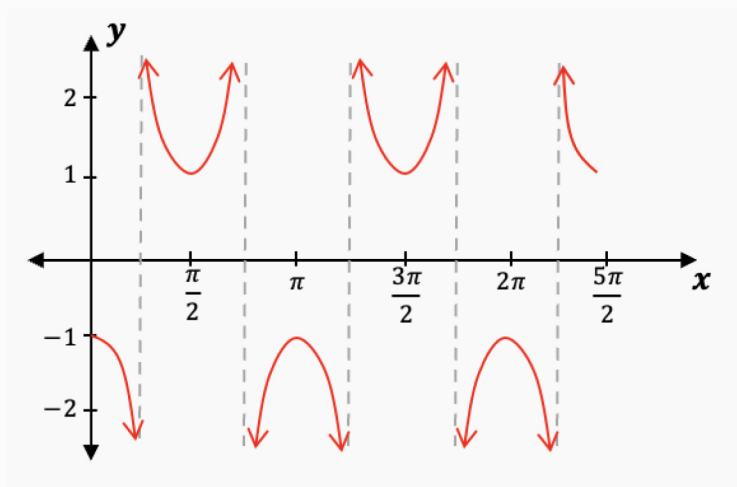
Graph the function $y = \csc(2x)$.



TOPIC: GRAPHS OF SECANT & COSECANT

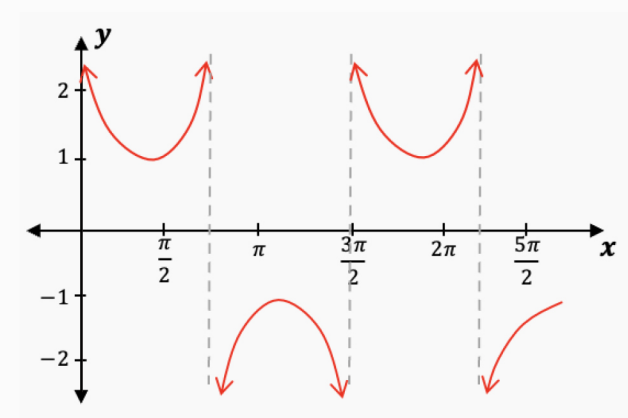
PRACTICE

Below is a graph of the function $y = \sec(bx - \pi)$. Determine the value of b .



PRACTICE

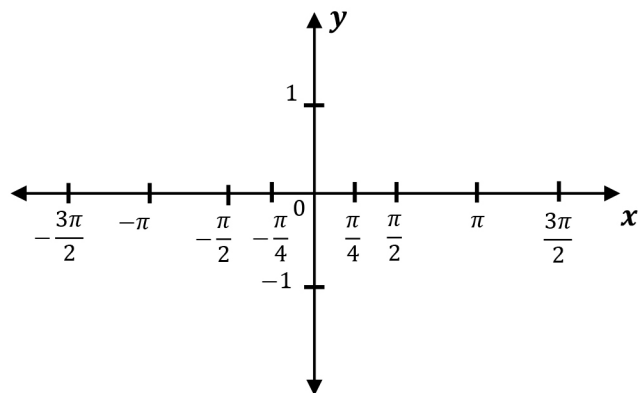
Below is a graph of the function $y = \csc(bx)$. Determine the value of b .



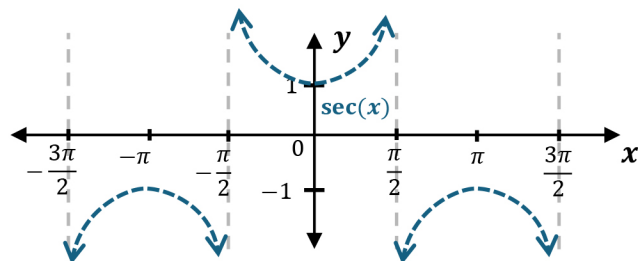
TOPIC: GRAPHS OF TANGENT & COTANGENT

Graphing the Tangent Function

◆ Recall: $\tan(x) = \frac{\sin(x)}{\cos(x)}$, so we can use the values of sin & cos to graph tan.



x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\sin x$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
$\cos x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0
$\tan x$					



► Like $\sec(x)$, $\tan(x)$ repeats and has asymptotes where $\cos(x) = 0$, at _____ multiples of _____.

► Unlike other trig functions, $\tan(x)$ has a period of _____. In general,

New

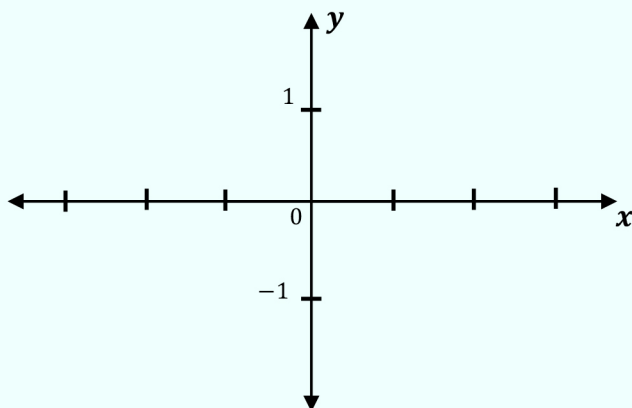
$y = \tan(bx) \rightarrow \text{Period } (T) = \text{_____}$

(Period of Tangent)

◆ You can use all of the same transformation rules you used for sin & cos to stretch and shift graphs of tan!

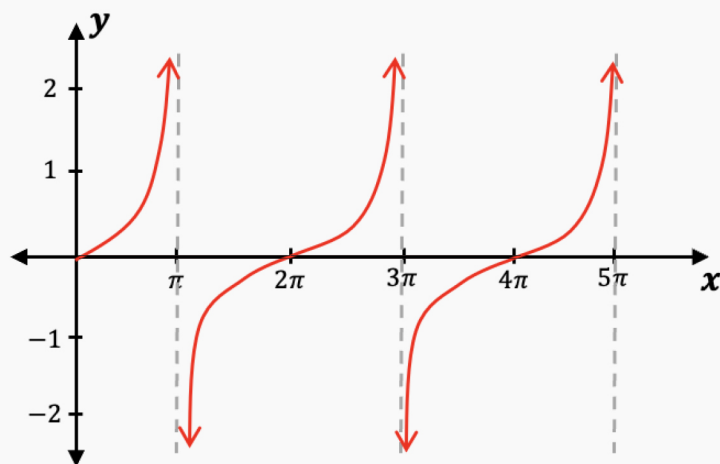
EXAMPLE

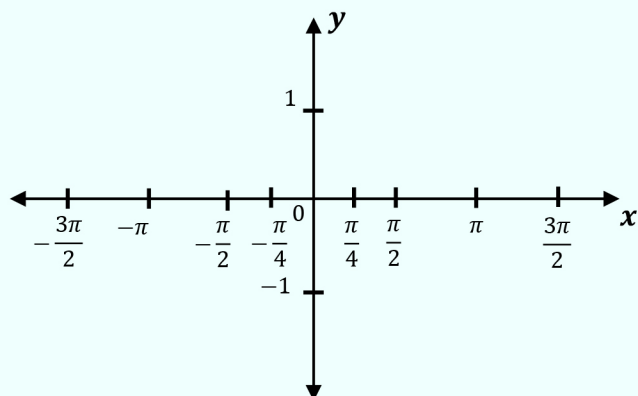
Graph the function $y = \tan\left(\frac{\pi}{2}x\right)$.



TOPIC: GRAPHS OF TANGENT & COTANGENT**PRACTICE**

Below is a graph of the function $y = \tan (bx)$. Determine the value of b .



TOPIC: GRAPHS OF TANGENT & COTANGENT**EXAMPLE**Graph the function $y = \frac{1}{2} \cdot \tan \left(x - \frac{\pi}{2} \right)$.

TOPIC: GRAPHS OF TANGENT & COTANGENT

Graphing the Cotangent Function

◆ We can use the graph of tan to graph cot.

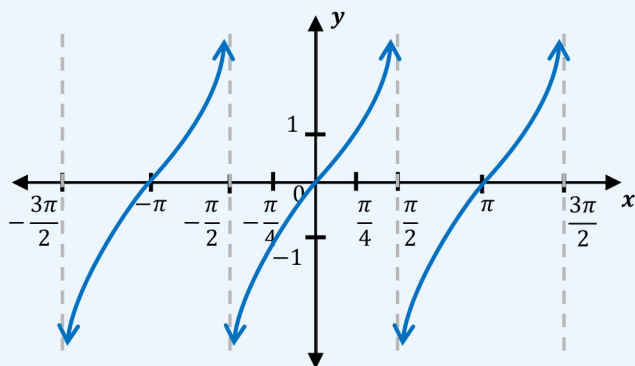
► **cot(x)** is similar to **tan(x)** but _____ upside-down, and has asymptotes at different values.

Recall Reciprocal Identities

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

Recall

Tangent



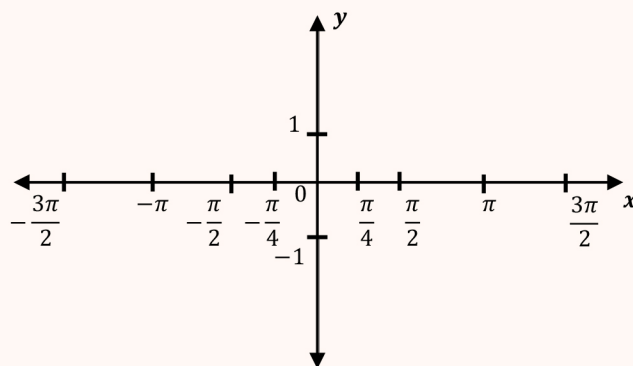
► Repeats every $[\pi \mid 2\pi]$

$$y = \tan(bx) \rightarrow \text{Period } (T) = \frac{\pi}{b}$$

► Asymptotes: **[odd | integer]** multiple of $\left[\frac{\pi}{2} \mid \pi\right]$

New

Cotangent



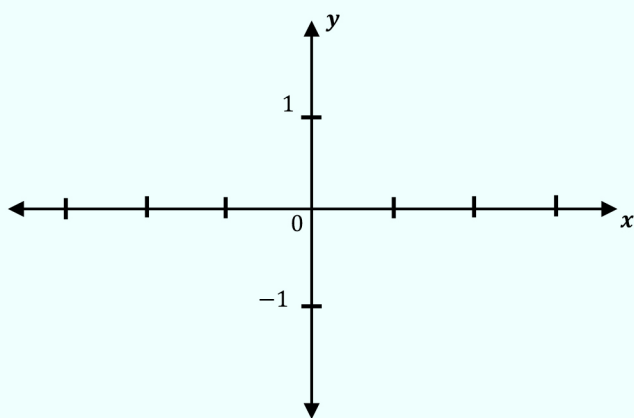
► Repeats every $[\pi \mid 2\pi]$

$$y = \cot(bx) \rightarrow \text{Period } (T) =$$

► Asymptotes: **[odd | integer]** multiple of $\left[\frac{\pi}{2} \mid \pi\right]$

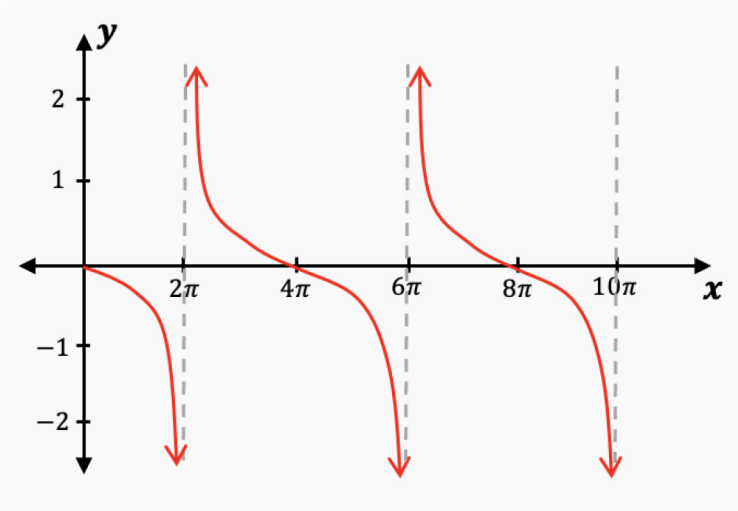
EXAMPLE

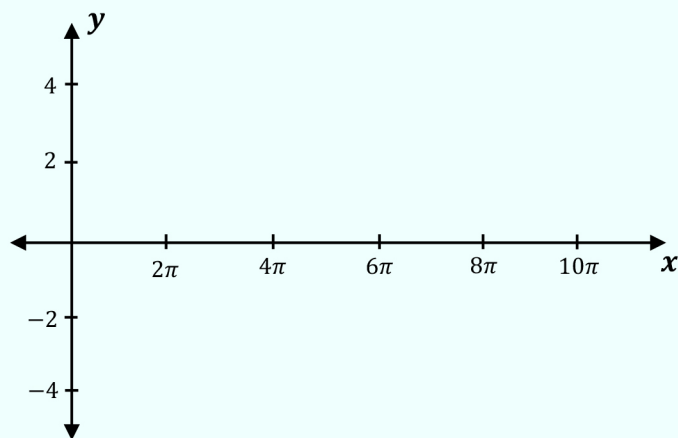
Graph the function $y = \cot(\pi x)$.



TOPIC: GRAPHS OF TANGENT & COTANGENT**PRACTICE**

Below is a graph of the function $y = \cot\left(bx + \frac{\pi}{2}\right)$. Determine the value of b .

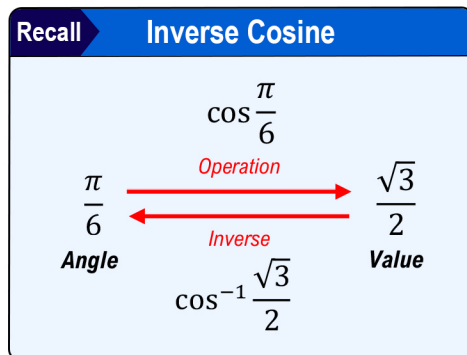


TOPIC: GRAPHS OF TANGENT & COTANGENT**EXAMPLE**Graph the function $y = -2 \cdot \cot\left(\frac{1}{4}x\right)$.

TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

Inverse Cosine

- ◆ Just like a square root *undoes* a square, the **inverse cosine** function (or arccos) *undoes* the cosine function.

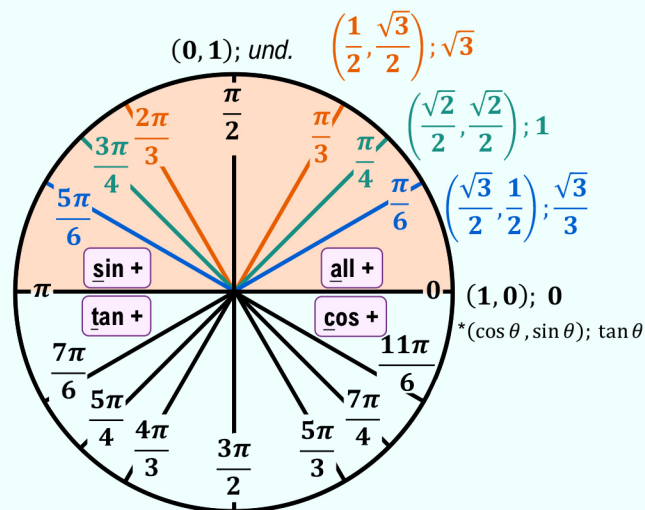


- ◆ To evaluate an $\cos^{-1}$ function, find the _____ with the corresponding value on the unit circle.

EXAMPLE

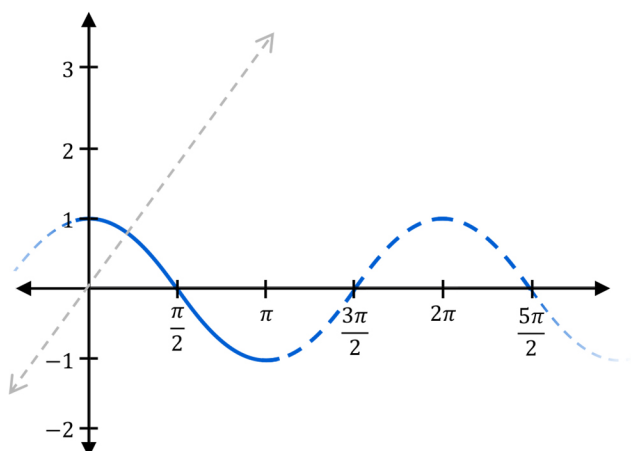
Evaluate the expression.

$$\cos^{-1} \frac{1}{2}$$



- ◆ Since $\cos(x)$ is not _____, $\cos^{-1}(x)$ is only defined for *certain values* so that it passes the vertical line test.

- Recall: An *inverse function* is a reflection over the line _____.
- So, when taking $\cos^{-1}(x)$, only use angles in the interval $[\text{____}, \text{____}]$.



New

Input: _____ $< x <$ _____

Output: _____ $< \theta <$ _____

(Inverse Cosine Interval)

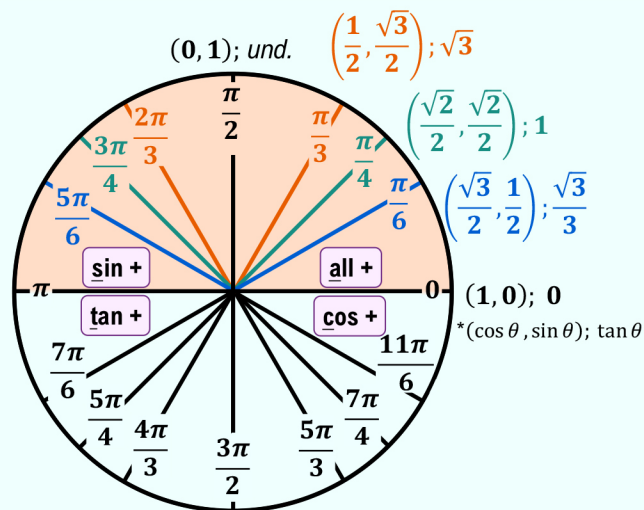


TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

EXAMPLE

Evaluate the expression.

$$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$



PRACTICE

Evaluate the expression.

(A) $\cos^{-1}(-1)$

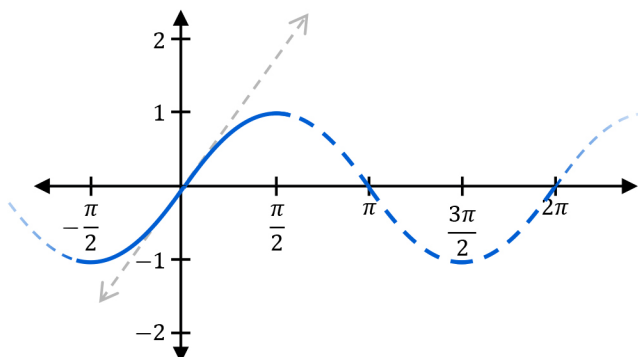
(B) $\cos^{-1} 0$

(C) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

Inverse Sine

- ◆ Like inverse cosine, $\sin^{-1}(x)$ is only defined for *certain values* because the sine function is not one-to-one.



New

Input: $____ < x < ____$

Output: $____ < \theta < ____$

(Inverse Sine Interval)

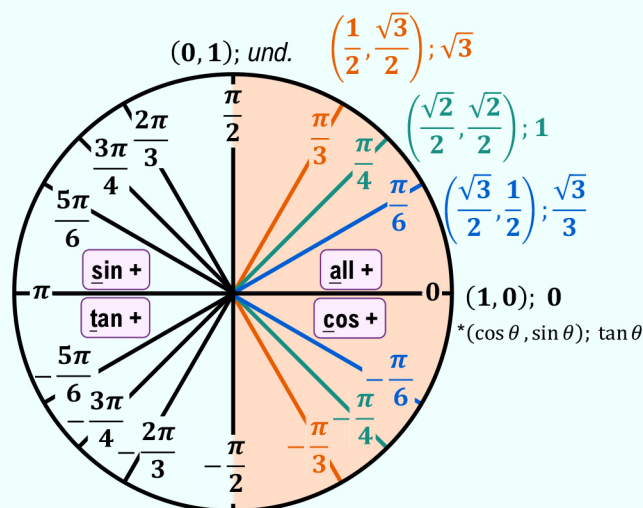
- ◆ To evaluate an inverse sine function, find the $\angle$ with the corresponding value on the unit circle within the interval.

EXAMPLE

Evaluate the expression.

(A) $\sin^{-1} \frac{1}{2}$

(B) $\sin^{-1} \left(-\frac{\sqrt{2}}{2} \right)$



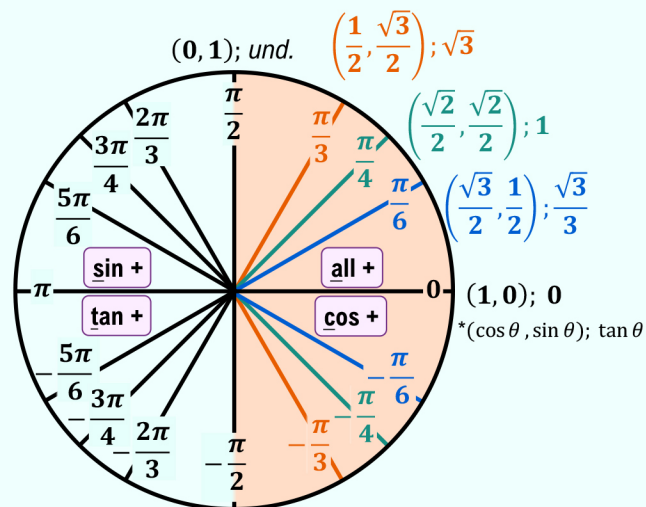


TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

EXAMPLE

Evaluate the expression.

$$\sin^{-1}(-1)$$



PRACTICE

Evaluate the expression.

(A) $\sin^{-1} 1$

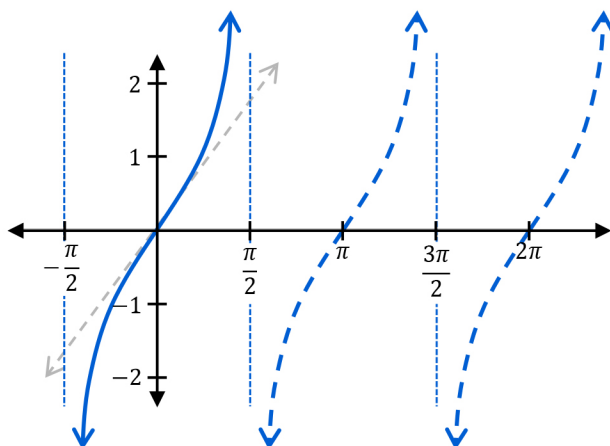
(B) $\sin^{-1} \frac{\sqrt{3}}{2}$

(C) $\sin^{-1} \left(\frac{\sqrt{2}}{2} \right)$

TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

Inverse Tangent

- ◆ Like inverse sine & cosine, $\tan^{-1}(x)$ is only defined for *certain values* because the tangent fn is not one-to-one.



New

Input: $\text{---} < x < \text{---}$

Output: $\text{---} < \theta < \text{---}$

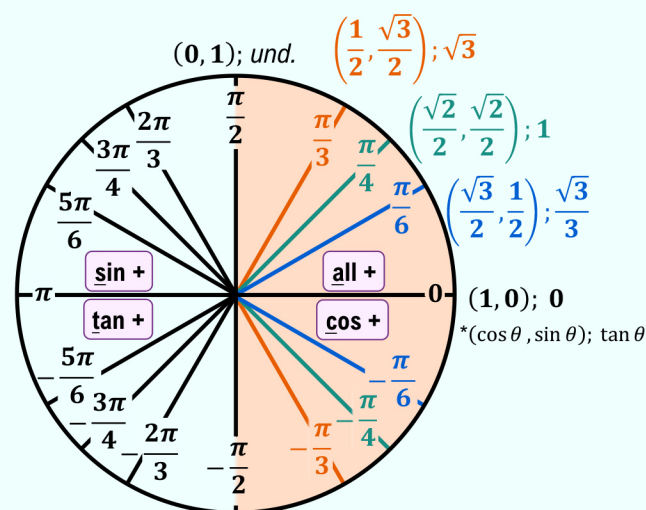
(Inverse Tangent Interval)

EXAMPLE

Evaluate the expression.

(A) $\tan^{-1} \sqrt{3}$

(B) $\tan^{-1}(-1)$



- ◆ To evaluate inverse trig functions using a calculator, press **2nd** → **SIN**, **COS** or **TAN**. (usually in RAD mode)

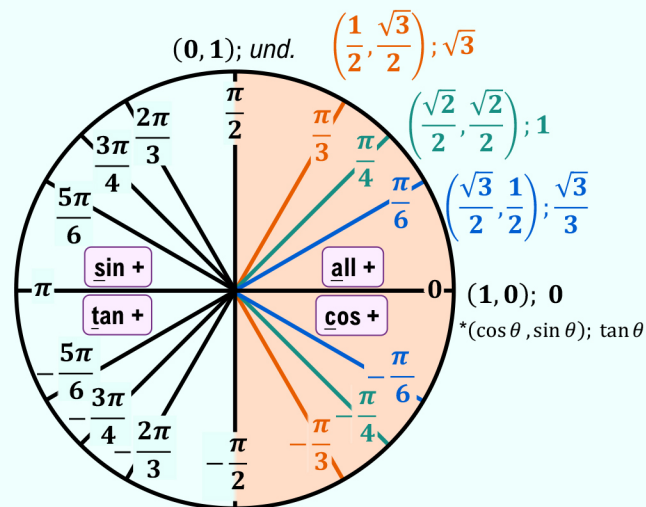


TOPIC: INVERSE TRIG FUNCTIONS & BASIC TRIG EQUATIONS

EXAMPLE

Evaluate the expression.

$$\tan^{-1}(-\sqrt{3})$$



PRACTICE

Evaluate the expression.

(A) $\tan^{-1} 0$

(B) $\tan^{-1} 1$

(C) $\tan^{-1} \left(-\frac{\sqrt{3}}{3} \right)$

PRACTICE

Evaluate the expression using a calculator. Express your answer in radians, rounding to two decimal places.

(A) $\tan^{-1}(5)$

(B) $\sin^{-1} \left(-\frac{1}{3} \right)$

(C) $\cos^{-1} \left(\frac{1}{4} \right)$