◆ **Polynomial** functions have a variable base with a number exponent; **exponential** functions have the opposite!

- Exponential functions have a:
  - **Base** that is __________, ___________, & ___ 1.
  - **Exponent** (power) that contains a __________.

Recall: Polynomial Function	New: Exponential Function
$f(x) = x^2$	$f(x) = 2^x$ (Power: x; Base: 2)

EXAMPLE Determine if each function is an exponential function.

(A) $f(x) = \left(\frac{2}{3}\right)^x$

Exponential function? ☐

Power: _______

Base: ________

(B) $f(y) = 1^y$

Exponential function? ☐

Power: _______

Base: ________

(C) $f(x) = 10^{x+1}$

Exponential function? ☐

Power: _______

Base: ________

◆ You will be asked to evaluate exponential functions for specified values of x.

- For exponents that cannot easily be done by hand, type (BASE) [ ] (POWER) into a calculator.

EXAMPLE Evaluate the exponential function $f(x) = 2^x$ for each given value of x.

(A) $x = 4$

(B) $x = -3$

(C) $x = 3.14$

(D) $x = 12$

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PRACTICE Determine if each function is an exponential function. If so, identify the power & base, then evaluate for $x = 4$.

(***A***)

$$f(x) = (-2)^x$$

Exponential function? ☐

Power: _______

Base: ________

$\boldsymbol{f(4)} =$_________

(***B***)

$$f(x) = 3(1.5)^x$$

Exponential function? ☐

Power: _______

Base: ________

$\boldsymbol{f(4)} =$_________

(***C***)

$$f(x) = \left(\frac{1}{2}\right)^x$$

Exponential function? ☐

Power: _______

Base: ________

$\boldsymbol{f(4)} =$_________

$f(x) = 2^x$

$f(x) = e^x$

◆ You will have to evaluate exponential expressions & graph exponential functions with a base of e:

- e, like π (3.14…), is ***NOT*** a variable, but a ***number***.

$$e = 2.718281828459\ldots$$

◆ A calculator should be used to evaluate an exponential function of base e: 2nd LN (POWER)

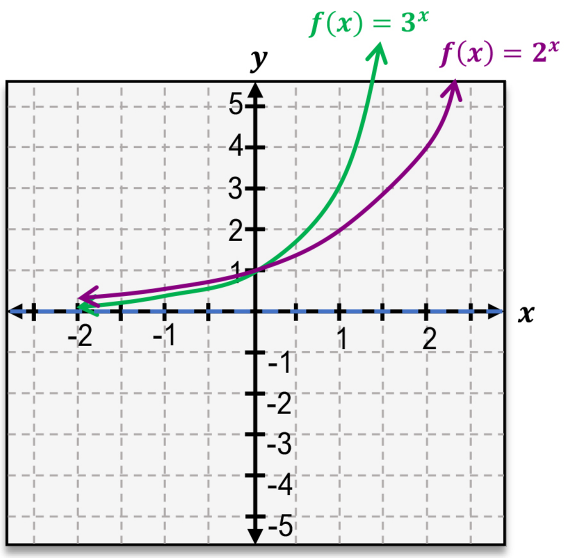
EXAMPLE Evaluate the exponential function $f(x) = e^x$ for each given value of x.

(***A***) $x = 2$

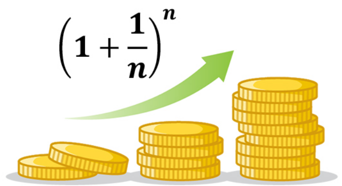
(***B***) $x = -3$

◆ The graph of $f(x) = e^x$ is __________ the graphs of $f(x) = 2^x$ and $f(x) = 3^x$ because 2 < ___ < 3.

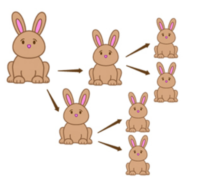
- Just like ***any*** exponential function, exponential functions of base e can be graphed using ________________.

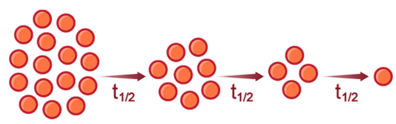


◆ e comes from the idea of compounding interest, but is useful in many different contexts.



Compounding Interest


Population Growth



Radioactive Decay

PRACTICE: Graph the given function.

$g(x) = e^{x+3} - 1$

TO GRAPH

0) Identify & graph parent function, $f(x) = b^x$
   a. Plot: $(-1, \underline{\;1/b\;})$, $(0,1)$, $(1, \underline{\;b\;})$, connect
   b. HA at: $y = 0$

1) Shift HA to $y = k$: $y = \_\_\_\_\_$

2) a. Reflect? □ → test points over $[x \mid y]$
   b. Shift test points by $(\underline{\;h\;}, \underline{\;k\;})$

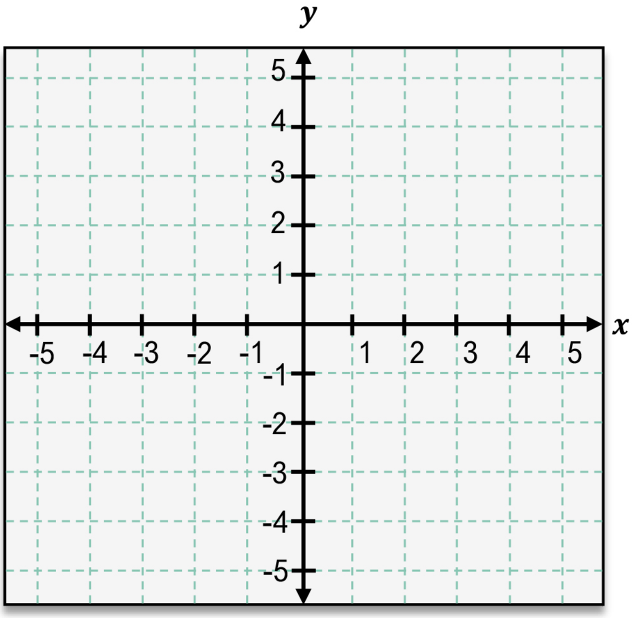
3) Sketch curve approaching asymptote

FROM GRAPH

Domain: *always* ________________

Range: If ABOVE asymp.: $(\underline{\;k\;}, \infty)$

If BELOW asymp.: $(-\infty, k)$



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◆ The _______________ (inverse) operation of an exponential is taking the ***logarithm*** (log).

- Logs and exponentials with the same **base** ________ each other.
- A **log** gives us the **power** that some **base** must be raised to in order to equal a particular number.

Solving Polynomials	Solving Exponentials		
$x^3 = 216$	*(A)* $2^x = 8$	*(B)* $2^x = 216$	*(Exponential Form)*
$\sqrt[3]{x^3} = \sqrt[3]{216}$	$2 \times 2 \times 2 = 8$		
$x = \sqrt[3]{216}$	$x = 3$	$x = \log$	*(Logarithmic Form)* *"log base 2 of 216"*

◆ You will need to convert expressions between **exponential form** and **logarithmic form**.

Recall: Exponential Form	New: Logarithmic Form
Equals $3^x = 81$ *Log base of*	$\log_{\square} \square = \square$

Exponential Form	Logarithmic Form
$\square^{\square} = \square$	*Equals* $x = \log_4 64$ *To the power of*

EXAMPLE Write each log in exponential form & each exponential in log form.

(A) $x = \log_5 800$

(B) $\log_2 16 = 4$

(C) $10^x = 4500$

◆ $\log_{10}$, known as the __________ log, can be written as just _____ and has its own calculator button: LOG

- Besides the common log, ($\log_{10}$), another frequently occurring log is $\log$___, called the _____________ log.
  - The natural log is written as ______, and also has its own calculator button: LN

Equals

$$b^x = M \quad \longrightarrow \quad \log_b M = x$$

Log base of

$$e^x = M \quad \longrightarrow \quad \underbrace{\log_{\square}}_{} M = x$$

EXAMPLE Write each log in exponential form & each exponential in log form.

(***A***) $x = \ln 17$

(***B***) $e^x = 4$

PRACTICE Convert the following logarithmic statement to its equivalent exponential form.

(***A***)

$$\log_4 x = 5$$

(***B***)

$$x = \log 9$$

PRACTICE Convert the following exponential statements to their equivalent logarithmic form.

(***A***)

$$3^x = 7$$

(***B***)

$$e^9 = x + 3$$

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◆ You can evaluate many logarithms using properties that come from the log being the _________ of an exponential.

New — **Properties of Logarithms**

Name	Example	Property	Description
Inverse Property	$\log_2 2^3 =$ ____ $2^{\log_2 3} =$ ____	$\log_b b^x = x$ $b^{\log_b x} = x$	Logs & exponentials w/ the *same base* ________
Log of the Base	$\log_2 2 =$ ___	$\log_b b = 1$	Log of its *base* equals ___
Log of 1	$\log_2 1 =$ ____ ______________ *"2 to what power gives 1?"*	$\log_b 1 = 0$	*ANY* log of 1 equals ___

EXAMPLE Using known properties, evaluate the given logarithms.

(**A**) $\log_2 \sqrt[3]{2}$

(**B**) $\ln 1$

(**C**) $\log 10$

(**D**) $\log_5 \frac{1}{5}$

PRACTICE Evaluate the given logarithm.

(***A***)

$$\log_7 7^{0.3}$$

(***B***)

$$\frac{3}{2}\log 1$$

(***C***)

$$\log_9 \frac{1}{81}$$

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◆ Properties of **exponents** correspond to properties of **logarithms** that we can use to expand log expressions.

New — **Properties of Logarithms**

Name	Exponents	Property		Description
Product Rule	$b^m \times b^n = b^{m+n}$	$\log_b(m \times n) = \log_b m$ ___ $\log_b n$		*Multiply* terms in a log → [ADD|SUBTRACT] logs
		EX.	$\log_2 3x =$	
Quotient Rule	$\frac{b^m}{b^n} = b^{m-n}$	$\log_b\left(\frac{m}{n}\right) = \log_b m$ ___ $\log_b n$		*Divide* terms in a log → [ADD|SUBTRACT] logs
		EX.	$\log_5 \frac{5}{y} =$	
Power Rule	$(b^m)^n = b^{m \cdot n}$	$\log_b m^n =$ ___$\log_b m$		Term to a *power* → __________ log by power
		EX.	$\ln 7^2 =$	

Expand & Condense Expressions Using Properties of Logs

◆ You may be given problems in which you are asked to either **expand** ***OR*** __________ logarithmic expressions.

- Properties of logarithms can be applied in _________ directions depending on your goal.

Properties of Logarithms

Name	Property
Product Rule	$\log_b(m \times n) = \log_b m + \log_b n$
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Rule	$\log_b m^n = n \log_b m$

Expand

Condense

EXAMPLE Expand the log expression as much as possible.

$\log_2 3xy^2$

EXAMPLE Condense the given expression into a single log.

$2\ln x - \ln(x+2)$

◆ When condensing logs, remember the base must be the ________ & always apply the power rule _________.

PRACTICE Write each logarithmic expression as a single log.

(***A***)

$$\log_2 \frac{1}{9x} + 2\log_2 3x$$

(***B***)

$$\ln \frac{3x}{y} + 2\ln 2y - \ln 4x$$

PRACTICE Write each single logarithm as a sum or difference of logs.

(***A***)

$$\log_3\left(\frac{\sqrt{x}}{9y^2}\right)$$

(***B***)

$$\log_5\left(\frac{5(2x+3)^2}{x^3}\right)$$

Evaluating Logarithms Using the Change of Base Property

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◆ If a logarithm doesn't have a base that is easy to evaluate, you can simply ___________ the base.

- You'll most often want to change the base to be either ____ or ____, to easily evaluate with any calculator.

New **Change of Base Property**

$$\log_b M = \frac{\log_a \quad}{\log_a \quad}$$

$$\log_b M = \frac{\log_{10} M}{\log_{10} b} = \underline{\qquad}$$

$$\log_b M = \frac{\log_e M}{\log_e b} = \underline{\qquad}$$

EXAMPLE Evaluate the given logarithms using the change of base property and a calculator. Use common logs for (***A***), (***B***) and natural logs for (***C***), (***D***).

(***A***) $\log_7 31 =$

(***B***) $\log_\pi 9 =$

(***C***) $\log_\pi 9 =$

(***D***) $\log_{\sqrt{3}} e =$

PRACTICE Evaluate the given logarithms using the change of base formula and a calculator. Use common logs.

(***A***)

$$\log_3 17$$

(***B***)

$$\log_9 67$$

PRACTICE Evaluate the given logarithms using the change of base formula and a calculator. Use natural logs.

(***A***)

$$\log_8 41$$

(***B***)

$$\log_2 3789$$

◆ We **evaluated** exp. functions for given x; we **solve** exp. *equations* for unknown to make the statement ________.

- To solve exp. equations, rewrite each side to have same _______ & set __________ equal, solving for ____.

Recall: Exponential *Function*	New: Exponential *Equation*
$f(x) = 2^x$	$16 = 2^x$
$f(4) = 2^4$	$2^{—} = 2^x$
$f(4) = 16$	$\_\_\_\_\_ = \_\_\_\_\_$

EXAMPLE

Solve each exponential equation by expressing each side as a power of the same base.

(***A***) $64 = 2^x$

(***B***) $5^{x+1} = \sqrt{5}$

(***C***) $27 = 9^x$

COMMON POWERS

Squares	Cubes
$2^2 = 4$	$2^3 = 8$
$3^2 = 9$	$3^3 = 27$
$4^2 = 16$	$4^3 = 64$
$5^2 = 25$	$5^3 = 125$
$6^2 = 36$	**Other**
$7^2 = 49$	
$8^2 = 64$	$2^4 = 16$
$9^2 = 81$	$2^5 = 32$
$10^2 = 100$	$3^4 = 81$

PRACTICE Solve the exponential equations.

(***A***) $4^{x+7} = 16$

(***B***) $100^{x} = 10^{x+17}$

(***C***) $81^{x+1} = 27^{x+5}$

◆ If you ***CANNOT*** rewrite sides of exponential equation to have the same base, solve using log or ln.

Like Bases	Unlike Bases
$16 = 2^x$	$17 = \quad 2^x$
$2^4 = 2^x$	$2^{???} = 2^x$
$4 = x$	

Properties of Logarithms

Name	Property
Product Rule	$\log_b(m \times n) = \log_b m + \log_b n$
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Rule	$\log_b m^n = n \log_b m$

EXAMPLE Solve each exponential equation by using logs.

(***A***) $10^x + 64 = 100$

(***B***) $3 = 2^{x+1}$

HOW TO: SOLVE EXP. EQNS WITH LOGS

1) ____________ exp. expression(s)
2) If base 10: Take _____ of both sides
If NOT base 10: Take ___ of both sides
3) Use log rules to get x out of exponent
4) __________ for x
5) (If asked) Approximate using calculator

◆ Recall: The log or ln of some number is a _____________, not a variable! Treat it like any other number.

PRACTICE Solve the exponential equations.

(***A***)

$$2 \cdot 10^{3x} = 5000$$

(***B***)

$$900 = 10^{x+17}$$

(***C***)

$$e^{2x+5} = 8$$

(***D***)

$$7^{2x^2-8} = 1$$

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◆ When solving log equations, you'll encounter two possibilities: $\log_b M = \log_b N$ **OR** $\log_b M = c$

- If a logarithmic equation can be written as $\log_b M = \log_b N$, set ____ = ____ & solve for $\boldsymbol{x}$.

Recall **Exponential Equation**	New **Logarithmic Equation**
$\mathbf{16 = 2^x}$	$\mathbf{\log_2(x+1) = \log_2 5}$
$\mathbf{2^4 = 2^x}$	______ = ____
$\mathbf{4 = x}$	______ = ____

EXAMPLE Solve the log equation.

$\ln(x+4) - \ln 2 = \ln 8$

Properties of Logarithms

Name	Property
Product Rule	$\log_b(m \times n) = \log_b m + \log_b n$
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Rule	$\log_b m^n = n \log_b m$

- If a log equation ***CANNOT*** be written as $\log_b M = \log_b N$ put in ______________ form, then solve for $\boldsymbol{x}$.

Recall:

Equals

$c = \log_b M$

To the power of

EXAMPLE Solve the log equation.

$\log_2(4x) = 5$

HOW TO:SOLVE LOG EQNS: EXP FORM

1) __________ log expression

2) Put in exponential form

3) _________ for $\boldsymbol{x}$.

4) _______ solution by plugging $\boldsymbol{x}$ in $\boldsymbol{M}$

- If $\boldsymbol{M} > \mathbf{0}$, DONE
- If $\boldsymbol{M} < \mathbf{0}$, _____ a solution

PRACTICE Solve the logarithmic equations.

(***A***)

$$\log_3(3x+9) = \log_3 5 + \log_3 12$$

(***B***)

$$\log(x+2) + \log 2 = 3$$

(***C***)

$$\log_7(6x+13) = 2$$