



Study Prep

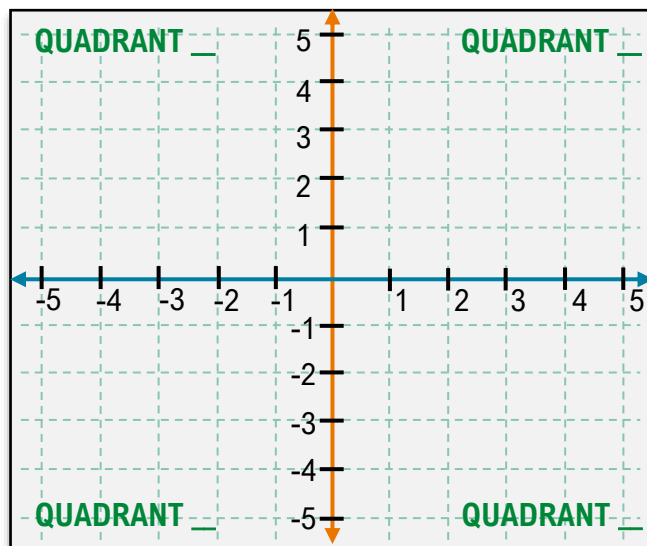
Available on Pearson+

**TOPIC: RECTANGULAR COORDINATE SYSTEM****Graphs & the Rectangular Coordinate System**

- Graphing in this course usually involves plotting _____ or _____ on the rectangular coordinate system.

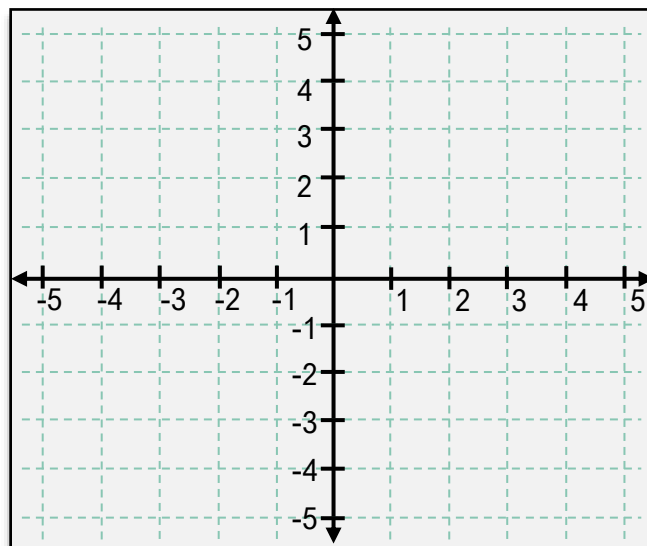
Rectangular Coordinate System ("Cartesian Plane"): 2 perpendicular _____ form a 2-D plane.

- Horizontal axis is the _____-axis
- Vertical axis is the _____-axis
- Ordered pairs / points:** Position *always* in form _____
- Origin:** point (____,____) where x & y axes intersect
 x values are [+ | -] [RIGHT | LEFT] of origin
 y values are [+ | -] [ABOVE | BELOW] origin
- Quadrants:** x & y axes divide graphs into 4 _____.
 Q1 starts at top-right, #s continue counter-clockwise



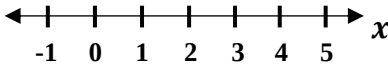
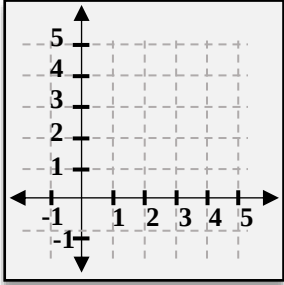
EXAMPLE: Plot the points $A(4, 3)$, $B(-3, 2)$, $C(-2, -3)$, $D(5, -4)$, $E(0, 0)$, $F(0, -3)$ on the graph above.

EXAMPLE: Graph the points $W(1, -2)$, $X(5, 2)$, $Y(-3, -4)$, $Z(-4, 3)$. Identify the **quadrant** of each point.



TOPIC: EQUATIONS OF TWO VARIABLES**Solving Two Variable Equations**

- Instead of just ONE variable, many equations in this course will involve TWO variables: ____ & ____

Equations with ONE Variable	Equations with TWO Variables
$x + 2 = 5$ $x = 3$  Solution: _____ point (x) on a 1D line	$x + y = 5$ $x = ? \quad y = ?$  Solution(s): _____ points _____ on a 2D plane

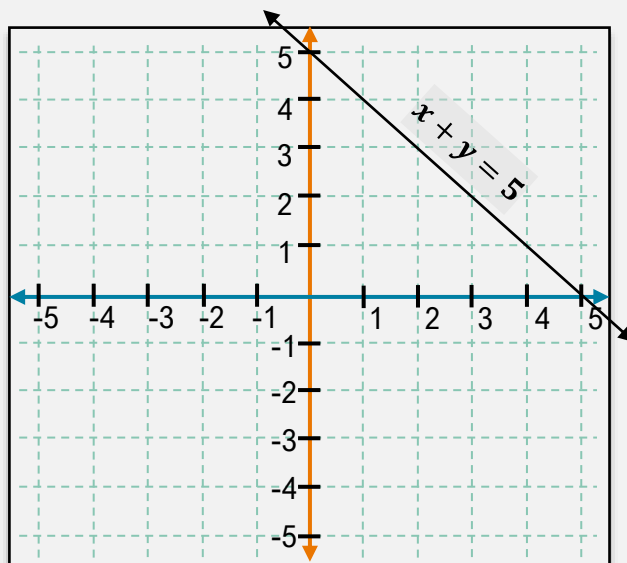
- If asked to determine if points (x, y) “satisfy” an equation, _____ x & y values to check if equation is true.

- The **graph** of an equation is just a visual representation of all (x, y) which make equation true.

When points **DO** satisfy an equation, they [**ARE | ARE NOT**] on the graph of that equation.

When points **DO NOT** satisfy an equation, they [**ARE | ARE NOT**] on the graph of that equation.

EXAMPLE: The graph of the equation $x + y = 5$ is a line, as shown in the diagram below. **a)** Determine if the points $(3, 2)$, $(4, 1)$, $(0, 0)$, & $(-1, 3)$ satisfy the equation. **b)** Plot each of the points on the graph.



**TOPIC: EQUATIONS OF TWO VARIABLES****Graphing Two Variable Equations by Plotting Points**

- To graph an equation, calculate & plot _____ that make the equation true.

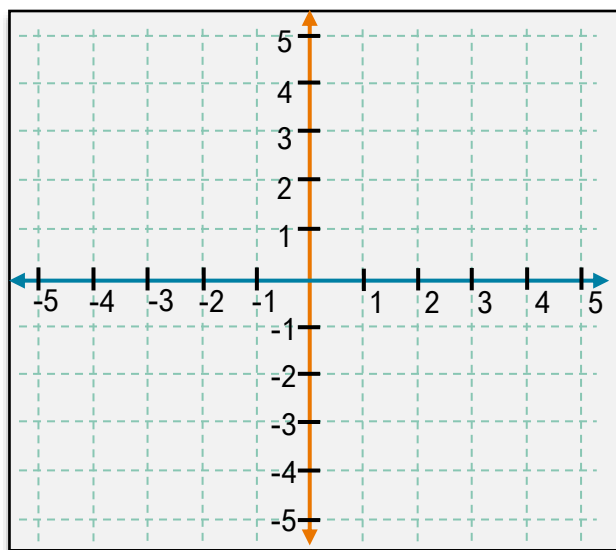
EXAMPLE: Graph the equation $-2x + y = -1$ by creating ordered pairs using $x = -2, -1, 0, 1, 2$.

x		y	Ordered pair (x, y)
-2			
-1			
0			
1			
2			

- If you're not *given* x -values to evaluate, choose your own!

GRAPHING BY PLOTTING POINTS

- 1) Isolate y to left side: $y = \dots$
- 2) Calculate y -values from 3-5 chosen x -values
- 3) Plot (x, y) points from Step 2
- 4) Connect points with line/curve





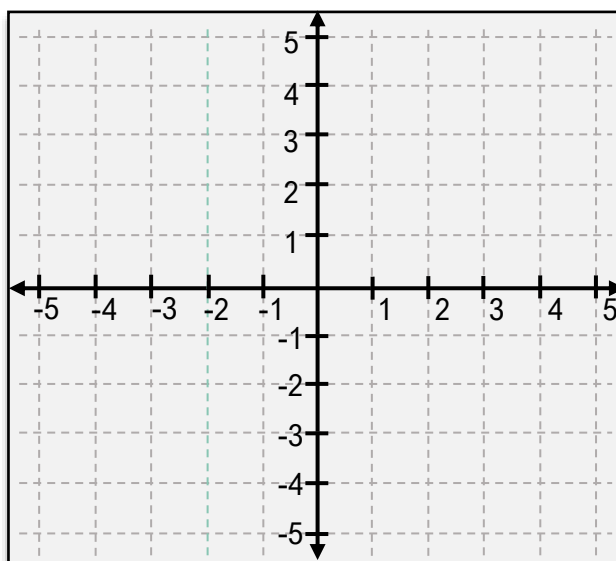
TOPIC: EQUATIONS OF TWO VARIABLES

PRACTICE: Graph the equation $y - x^2 + 3 = 0$ by choosing points that satisfy the equation.

x		y	Ordered Pair (x, y)

GRAPHING BY PLOTTING POINTS

- 1) Isolate y to left side: $y = \dots$
- 2) Calculate y -values from 3-5 chosen x -values
- 3) Plot (x, y) points from Step 2
- 4) Connect points with line/curve





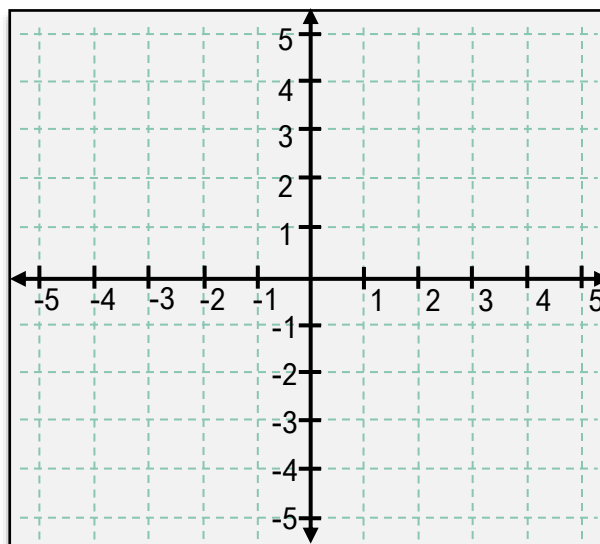
TOPIC: EQUATIONS OF TWO VARIABLES

PRACTICE: Graph the equation $y = \sqrt{x} + 1$ by choosing points that satisfy the equation. (*Hint: Choose positive numbers only*)

x		y	Ordered Pair

GRAPHING BY PLOTTING POINTS

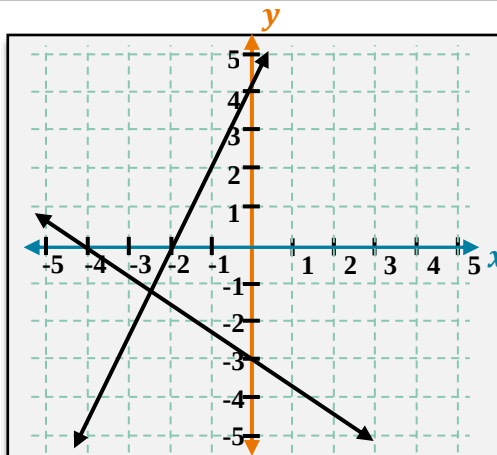
- 1) Isolate y to left side: $y = \dots$
- 2) Calculate y -values from 3-5 chosen x -values
- 3) Plot (x, y) points from Step 2
- 4) Connect points with line/curve



**TOPIC: EQUATIONS OF TWO VARIABLES****Graphing Intercepts**

INTERCEPTS: Ordered pairs/point(s) where graph crosses x or y axis

x – intercept



y – intercept

x – Intercept

x – intercept: x value when graph crosses $[x | y]$ axis

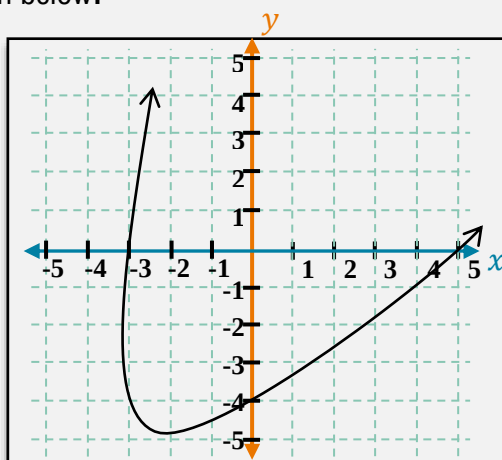
$[x | y]$ value is ALWAYS zero

y – Intercept

y – intercept: y value when graph crosses $[x | y]$ axis

$[x | y]$ value is ALWAYS zero

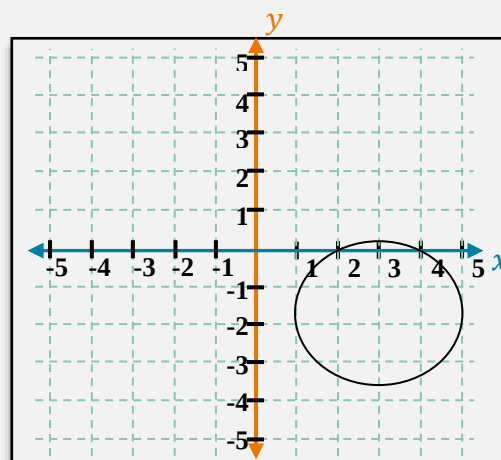
EXAMPLE: Write the x -intercepts & y -intercepts of the graph below.



x – intercept: _____

y – intercept: _____

EXAMPLE: Find the intercepts of the graph below.



Intercepts: _____

- If asked for “ x - or y -intercept”, simply write the x or y -value. If asked for just “intercepts”, write the ordered pairs.

TOPIC: LINES**Slopes of Lines**

- **Slope:** A number measuring how _____ a line is; how much **y changes** divided by how much **x changes**.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

Δ = "change in"

(x_1, y_1) & (x_2, y_2) are 2 points, may be given or chosen

EXAMPLE: Find the slopes of lines A & B shown in the graph.

Line A

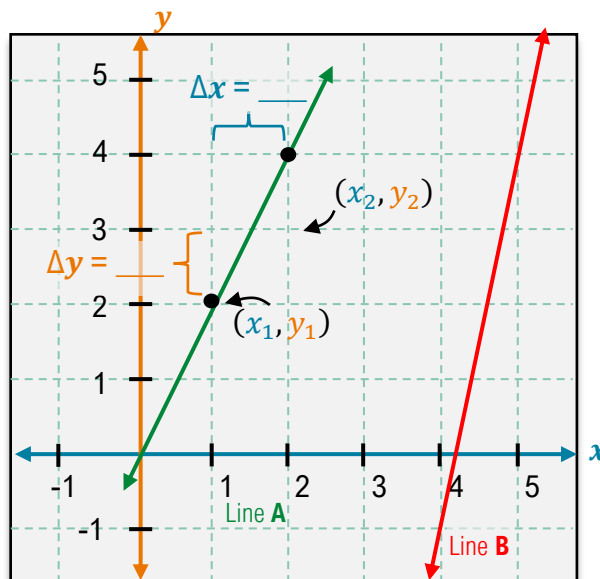
$$(x_1, y_1) = \underline{\hspace{2cm}}$$

$$(x_2, y_2) = \underline{\hspace{2cm}}$$

Line B

$$(x_1, y_1) = \underline{\hspace{2cm}}$$

$$(x_2, y_2) = \underline{\hspace{2cm}}$$



Note: Order of points doesn't matter, slope will not change. Generally, set (x_1, y_1) as the _____ point.

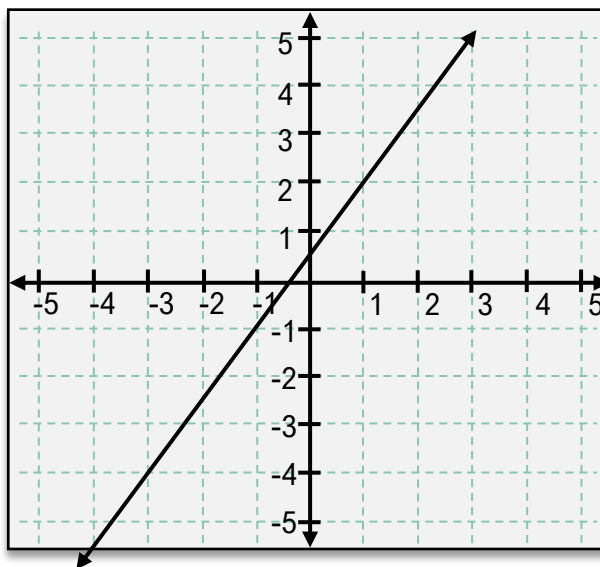
EXAMPLE: Find the slope of **Line B** above using $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (1, 2)$ instead.

**TOPIC: LINES**

PRACTICE: Find the slope of the line shown below.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

(Slope)



PRACTICE: Find the slope of the line containing the points $(-1, 1)$ and $(4, 3)$.

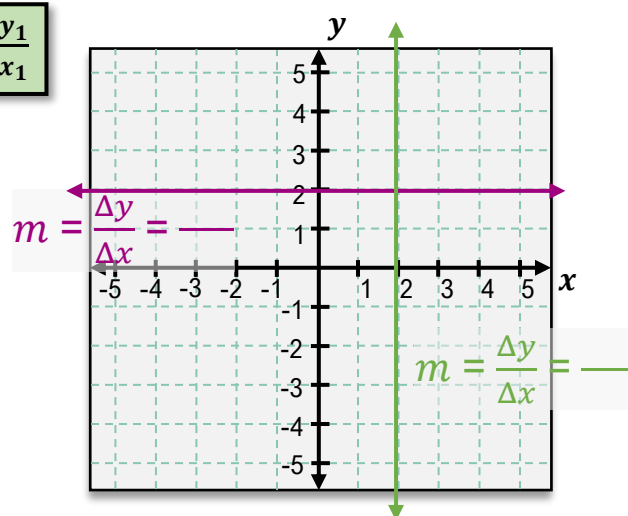
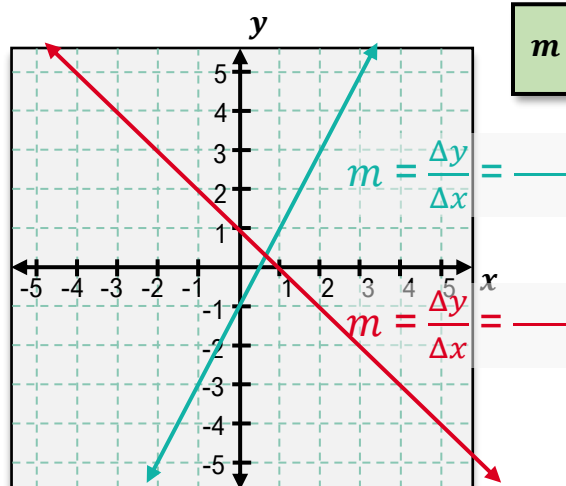
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

(Slope)

TOPIC: LINES

Types of Slope

- Slope can be **positive**, **negative**, **zero**, or **undefined**.



If line goes **UP** from left to right, slope is [+ | -]

If line goes **DOWN** from left to right, slope is [+ | -]

HORIZONTAL line: slope is [0 | UNDEFINED] $y = \#$

VERTICAL line: slope is [0 | UNDEFINED] $x = \#$

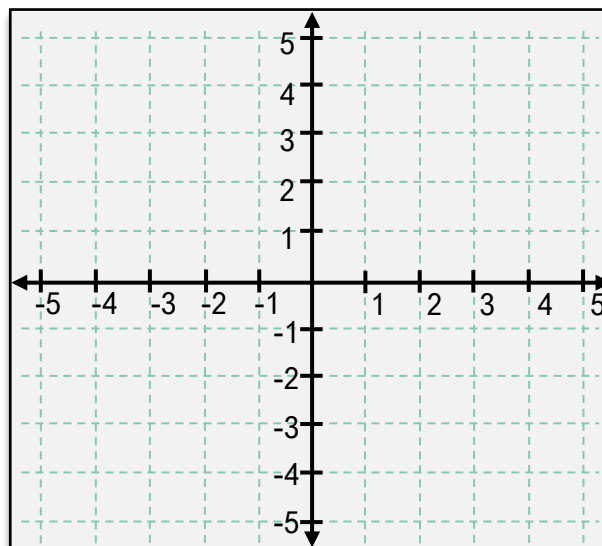


TOPIC: LINES

PRACTICE: Graph a line with a slope of 0 that passes through the point $(3, -2)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

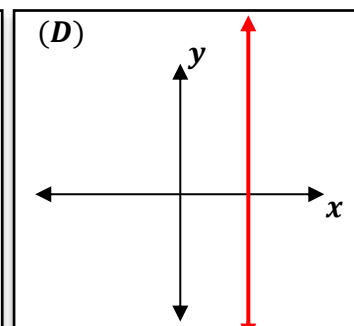
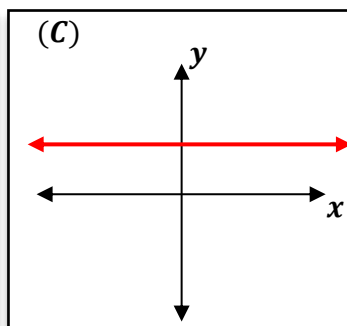
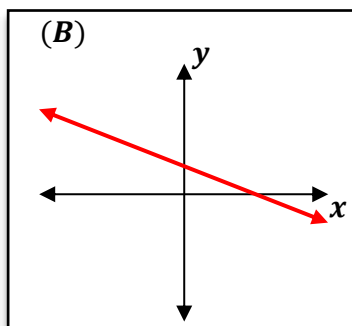
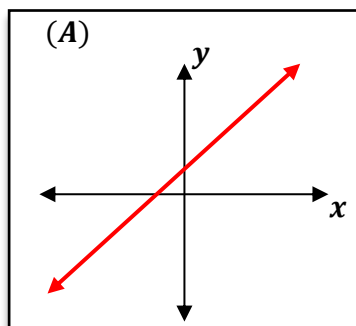
(Slope)



PRACTICE: Which of the following graphs below represents the equation $x = 3$?

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

(Slope)

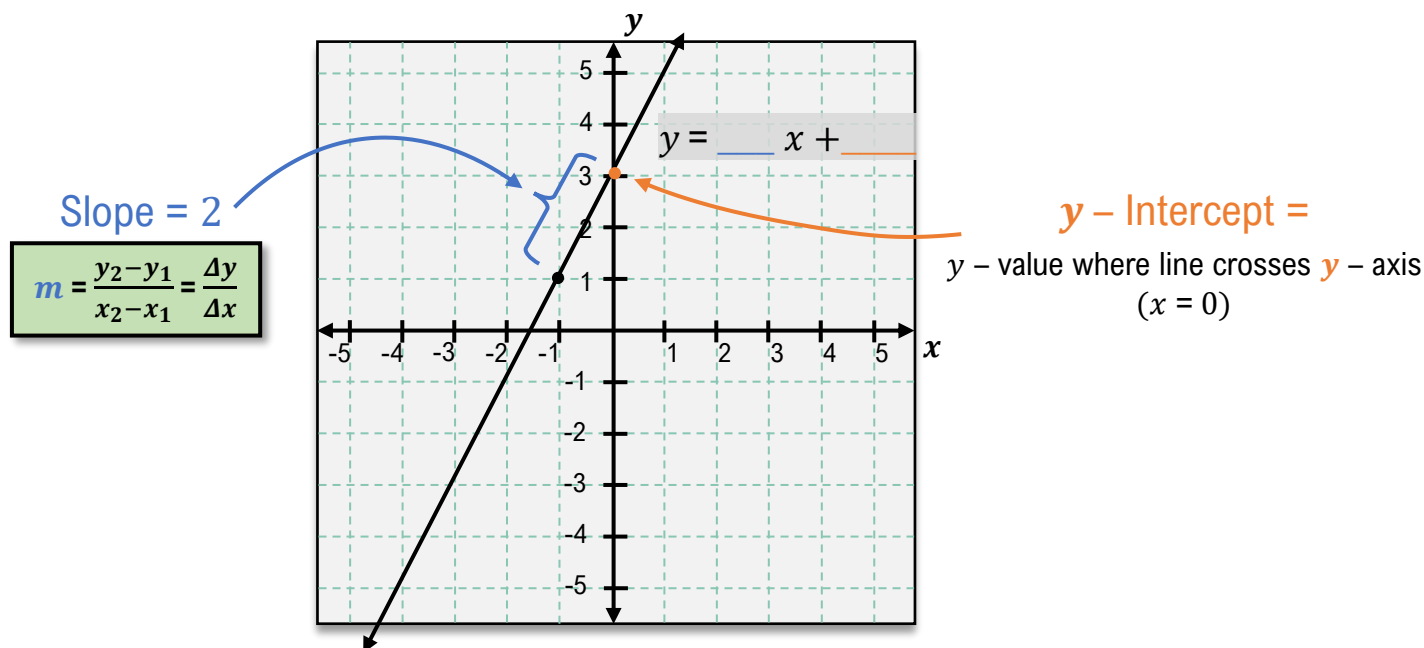


**TOPIC: LINES****Slope – Intercept Form**

- We can write the equation of a line using its _____ & _____.

$$y = \underline{\hspace{1cm}}x + \underline{\hspace{1cm}}$$

(Slope – Intercept Form)

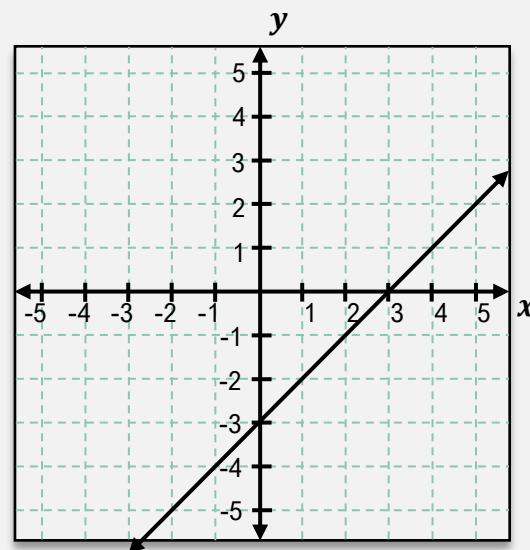


EXAMPLE: In the graph below, identify the y – intercept & slope.

Write the equation in slope-intercept form.

$$b = \underline{\hspace{1cm}}$$

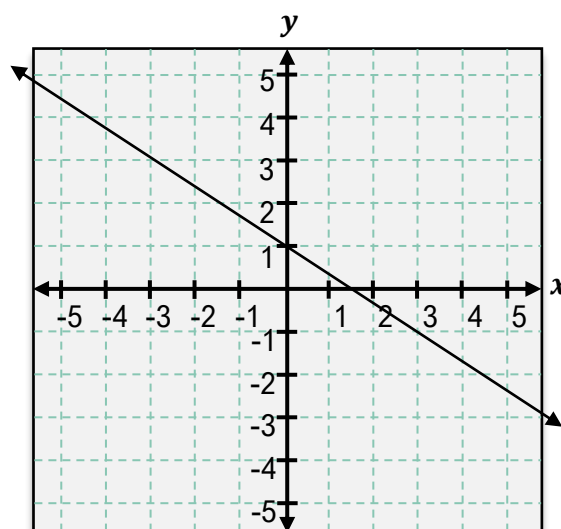
$$m = \underline{\hspace{1cm}}$$



**TOPIC: LINES**

PRACTICE: In the graph shown, identify the y – intercept & slope. Write the equation of this line in Slope-Intercept form.

$$y = mx + b$$



**TOPIC: LINES****Graphing Lines from Equations in Slope-Intercept Form**

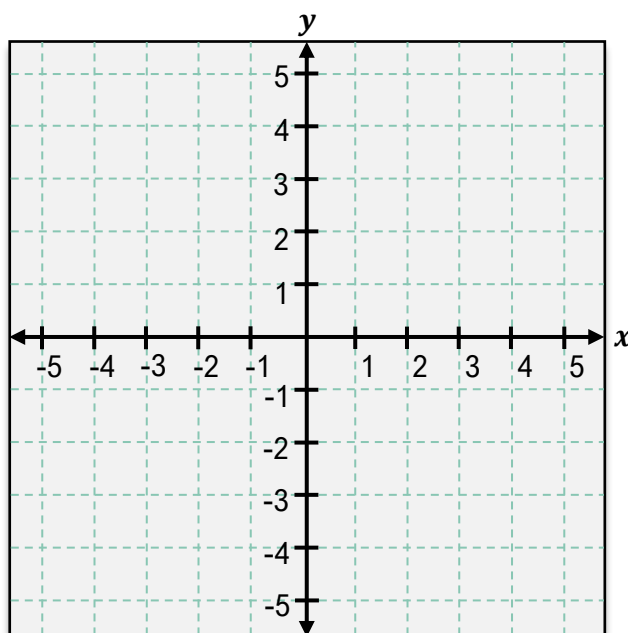
- A line equation in slope-intercept form tells you everything you need to graph it!

$$y = mx + b$$

EXAMPLE: Identify the **y – intercept** & **slope** of $y = \frac{2}{3}x + 1$, then graph the equation.

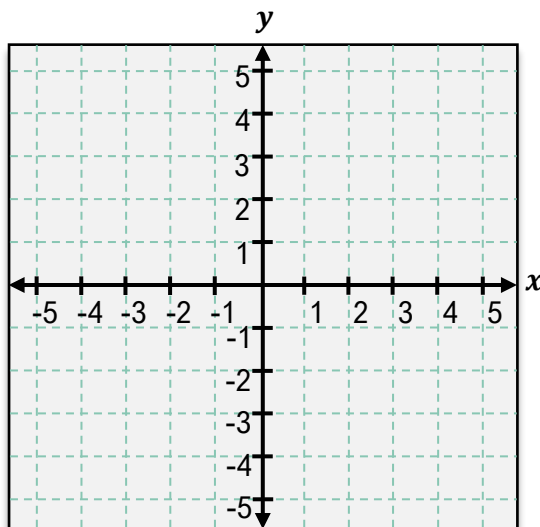
$$b = \underline{\hspace{2cm}}$$

$$m = \underline{\hspace{2cm}}$$

**Graphing Lines in Slope-Intercept Form**

- 1) Plot **y – intercept** $(0, b)$
- 2) Plot ONE more point using **slope** $\left(\frac{\text{rise}}{\text{run}}\right)$:
- 3) Connect points with a line

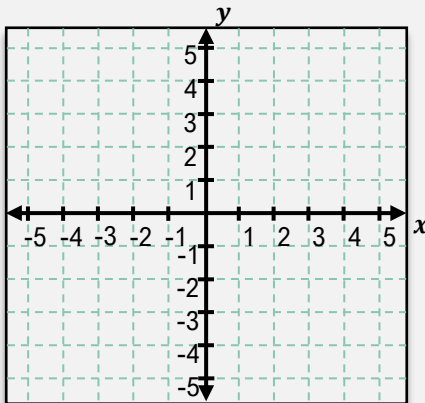
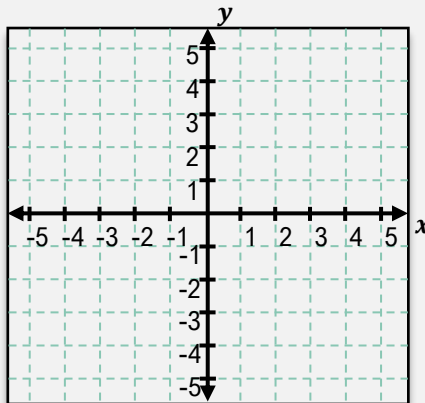
PRACTICE: Identify the **y – intercept** & **slope** of $y = -2x - 3$. Then graph the equation.



TOPIC: LINES

Point – Slope Form

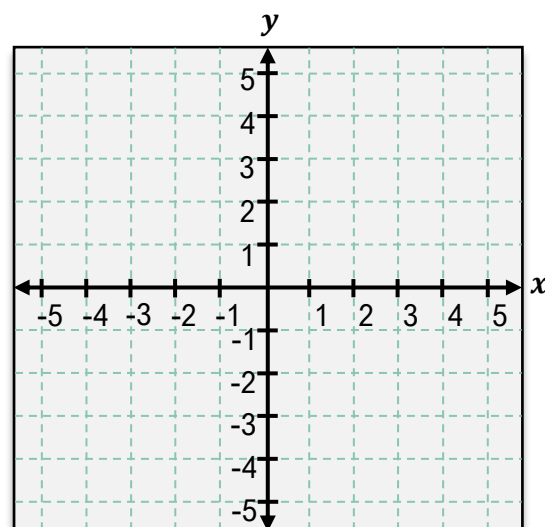
- If asked to write the equation of a line passing through a **point** that is _____ the **y-intercept**, use **Point-Slope** Form.

FORMS OF LINEAR EQUATIONS		
	SLOPE-INTERCEPT	POINT-SLOPE
EQN	$y = mx + b$	$y - y_1 = m(x - x_1)$
USE IF	Given m & b , asked for graph/EQ Given graph/EQ, asked for m & b	Given m & point (x_1, y_1) OR Given 2 points (x_1, y_1) & (x_2, y_2)
EXAMPLE	Graph the equation $y = \frac{2}{3}x - 1$ 	a) Write the equation of a line in point – slope form with slope $= \frac{2}{3}$ which passes through (3,1). b) Graph the line c) Rewrite your equation from part (a) in Slope-Intercept form 

PRACTICE: Write the point-slope form of the equation of a line with a slope of $-\frac{2}{5}$ that passes through $(1,3)$. Then graph the equation.

$$y - y_1 = m(x - x_1)$$

(Point-Slope)

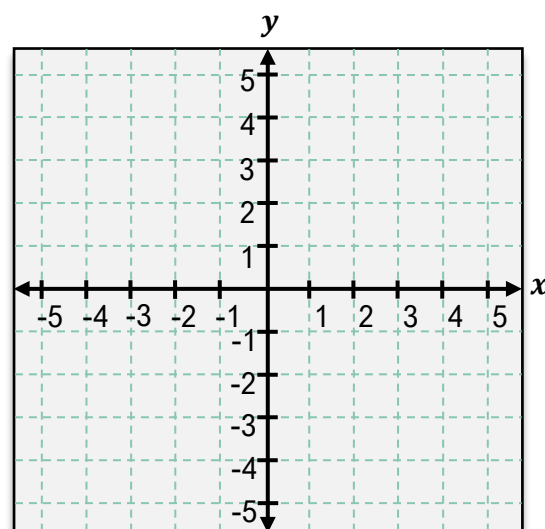


**TOPIC: LINES**

PRACTICE: Write the point-slope form of the equation of a line with a slope of 0 that passes through $(2, -4)$. Then graph the equation.

$$y - y_1 = m(x - x_1)$$

(Point-Slope)





TOPIC: LINES

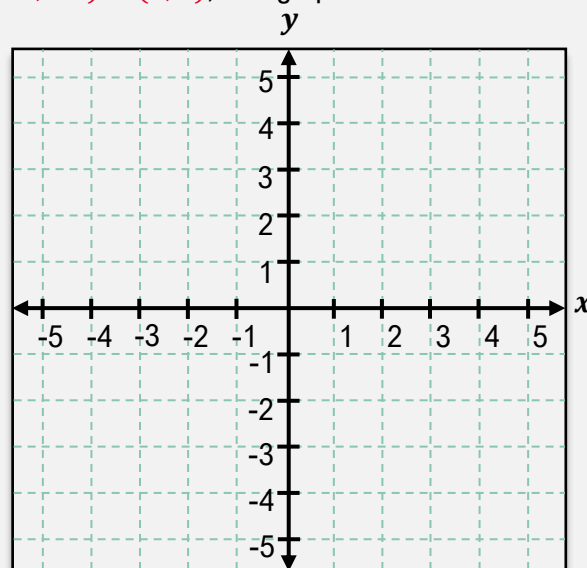
Finding Equations of a Line Given 2 Points

- Sometimes you will not be given slope, and you'll be asked to write an equation when given TWO points.

▪ Use _____ of the two points as (x_1, y_1)

FORMS OF LINEAR EQUATIONS		
	SLOPE-INTERCEPT	POINT-SLOPE
EQN	$y = mx + b$	$y - y_1 = m(x - x_1)$
USE IF	Given m & b , asked for graph/EQ Given graph/EQ, asked for m & b	Given m & point (x_1, y_1) OR Given 2 points (x_1, y_1) & (x_2, y_2)

EXAMPLE: Write the equation of the line passing through the points $(-1, -5)$ & $(2, 4)$, and graph it.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

(Slope)

$$y - y_1 = m(x - x_1)$$

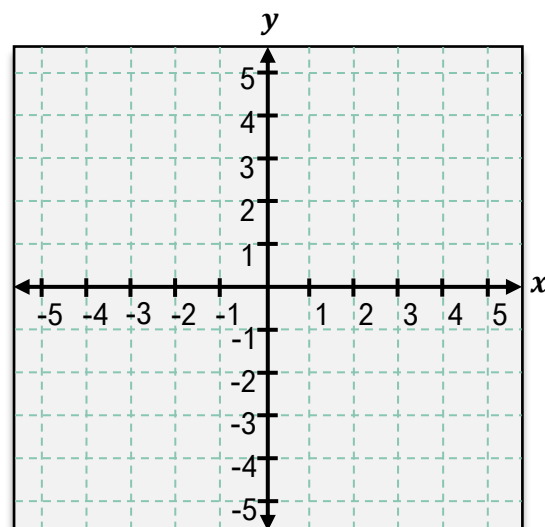
(Point - Slope Form)

**TOPIC: LINES**

PRACTICE: Write the point-slope form of the equation of a line that passes through the points (2,1) and (−4,3). Then graph the equation.

$$y - y_1 = m(x - x_1)$$

(Point-Slope)



**TOPIC: LINES****Standard Form of a Line**

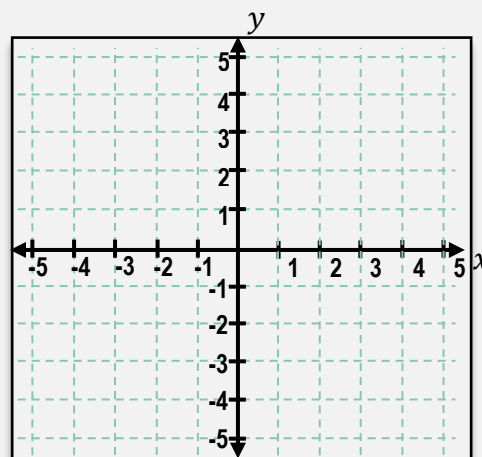
- When problems give a line in **Standard** Form and ask for **Slope** or **Intercept**, you'll have to _____ it.
 - To do this, _____ y to left side of equation.

FORMS OF LINEAR EQUATIONS			
SLOPE-INTERCEPT		POINT-SLOPE	STANDARD or “GENERAL”
EQN	$y = mx + b$ $y = 2x + 3$	$y - y_1 = m(x - x_1)$	$Ax + By + C = 0$ $-4x + 2y - 6 = 0$
USE IF	Given/asked for b	Given (x_1, y_1) & m or (x_2, y_2)	1) Asked to rewrite in other form <u>OR</u> 2) Finding x & y _____

EXAMPLE: Find the **slope** & **y-intercept** of the equation $-9x + 3y - 12 = 0$.

- To graph a line in Standard Form, you can find the x & y intercepts quickly *without* rewriting in Slope-Intercept.
 - For **x-intercept**, set $y = 0$ & solve for x . For **y-intercept**, set $x = 0$ and solve for y .

EXAMPLE: Graph the equation $3x + 2y - 6 = 0$ by finding the intercepts without using Slope-Intercept form.

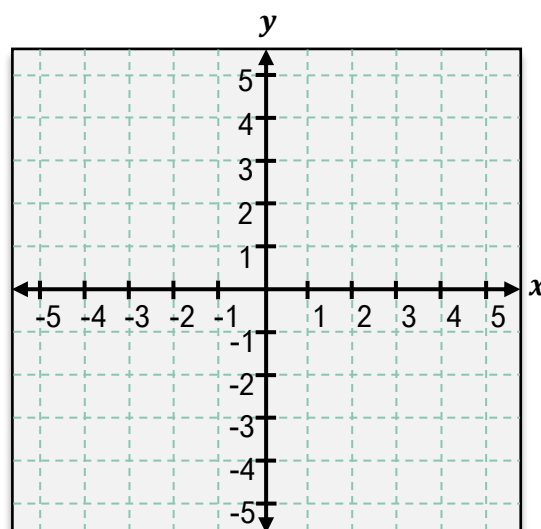


TOPIC: LINES

PRACTICE: Find the **slope** & **y-intercept** of the line given by the equation $3x + 2y - 6 = 0$.

FORMS OF LINEAR EQUATIONS		
SLOPE-INTERCEPT	POINT-SLOPE	STANDARD
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By + C = 0$

PRACTICE: Graph the equation $9x + 6y + 18 = 0$ by finding the intercepts.



TOPIC: LINES

Parallel and Perpendicular Lines

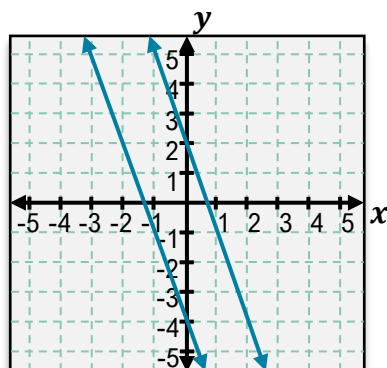
- Parallel & perpendicular lines are related by _____.

FORMS OF LINEAR EQUATIONS		
SLOPE-INTERCEPT	POINT-SLOPE	STANDARD
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By + C = 0$

Parallel Lines

$$y = -3x + 2$$

$$y = -3x - 4$$



$$m_1 = m_2$$

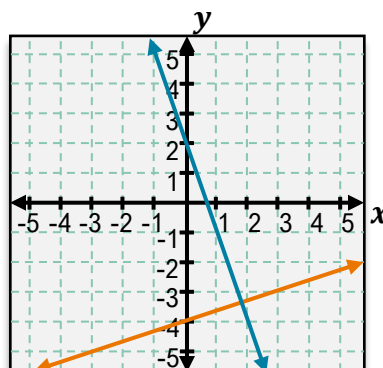
Slopes are _____; y-intercepts are _____

Lines **never** intersect

Perpendicular Lines

$$y = -3x + 2$$

$$y = \frac{1}{3}x - 4$$

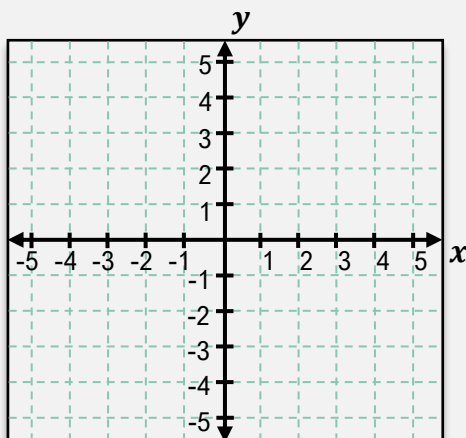


$$m_1 = -\frac{1}{m_2}$$

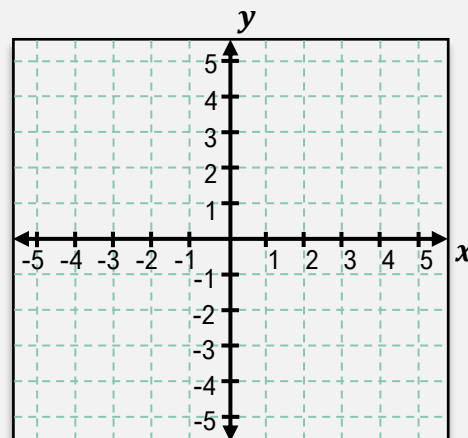
Slopes have _____ signs and are _____

Lines intersect at **right** angles (90°)

EXAMPLE: Write the equation of a line passing through $(-1, 4)$ that is parallel to $y = 2x - 6$.



EXAMPLE: Write the equation of a line perpendicular to $x + 4y - 8 = 0$ that has a y-intercept of 3.



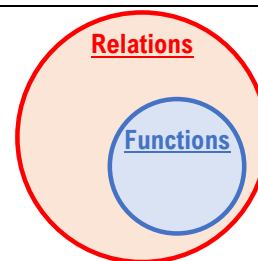


TOPIC: LINES

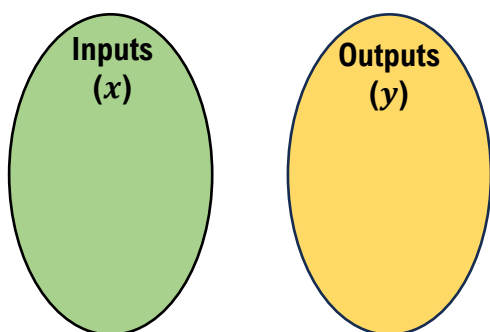
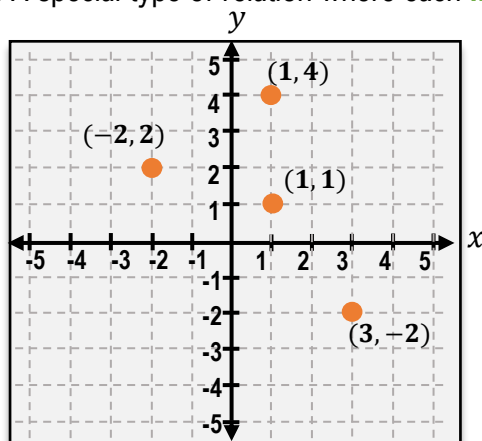
PRACTICE: Write an equation of a line that passes through the point $(3, -4)$ and is parallel to the line $x + 2y + 18 = 0$.

TOPIC: INTRO TO FUNCTIONS & THEIR GRAPHS

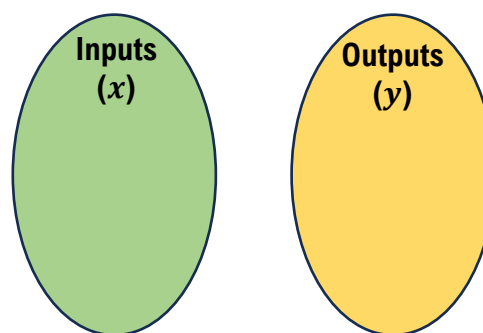
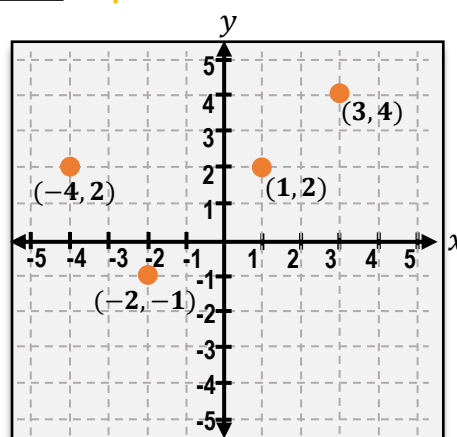
Relations and Functions



- **Relation**: A connection between _____ & _____ values.
 - Graphically, they are represented as _____ pairs (x, y)
- **Function**: A special type of relation where each **input** has at most _____ **output**.

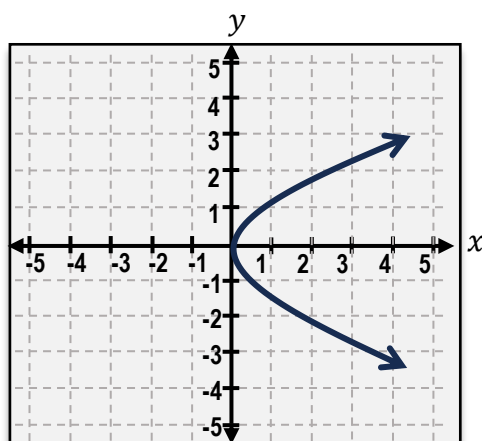


[FUNCTION | NOT A FUNCTION]

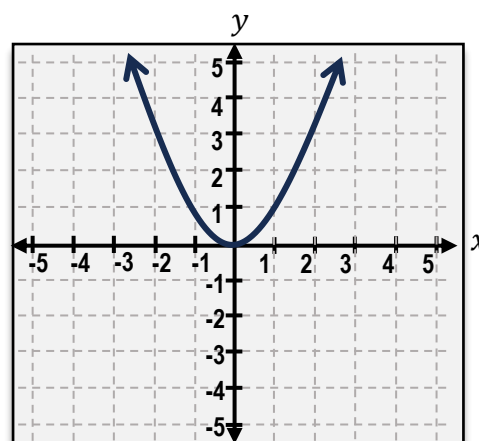


[FUNCTION | NOT A FUNCTION]

- A way to quickly determine if a graph is a function or not is the **Vertical Line Test**:
 - If you can draw **any** vertical line that passes through more than 1 point, the graph _____ a function.



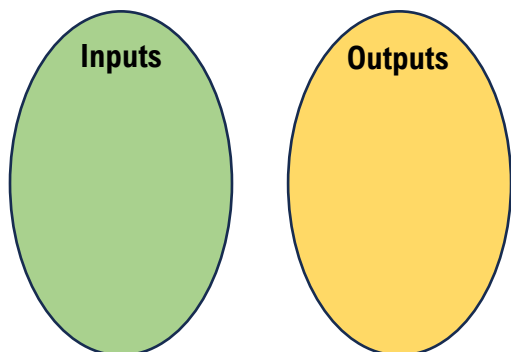
[FUNCTION | NOT A FUNCTION]



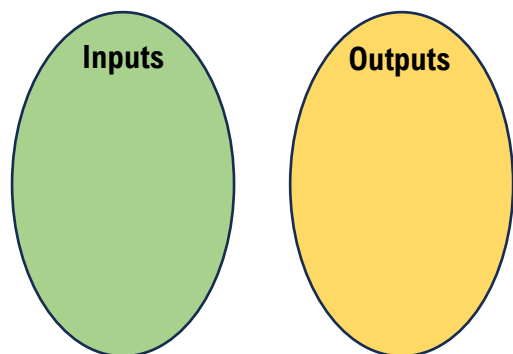
[FUNCTION | NOT A FUNCTION]

TOPIC: INTRO TO FUNCTIONS & THEIR GRAPHS

PRACTICE: State the inputs and outputs of the following relation. Is it a function? $\{(-3, 5), (0, 2), (3, 5)\}$

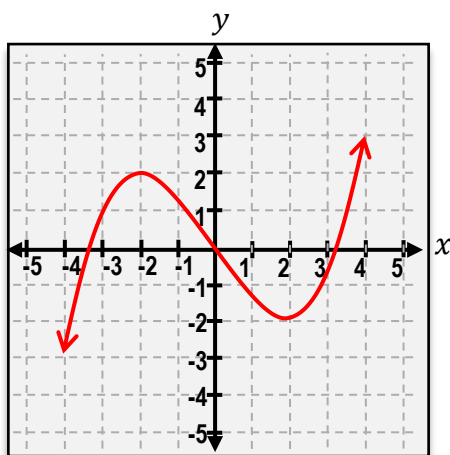


PRACTICE: State the inputs and outputs of the following relation. Is it a function? $\{(2, 5), (0, 2), (2, 9)\}$

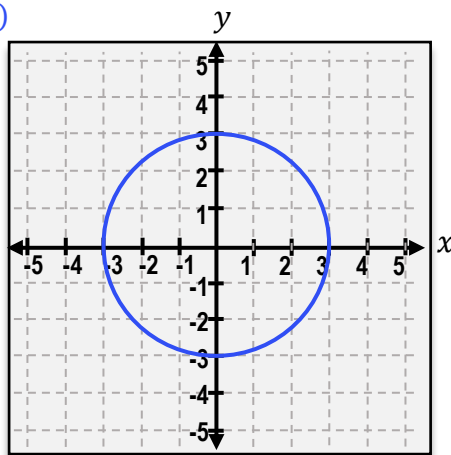


PROBLEM: Determine below which of the graphs are functions (select all that apply).

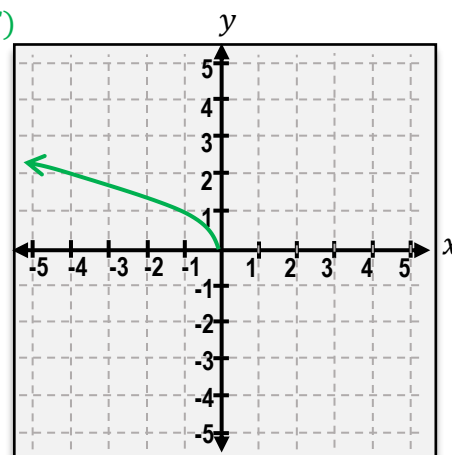
(A)



(B)

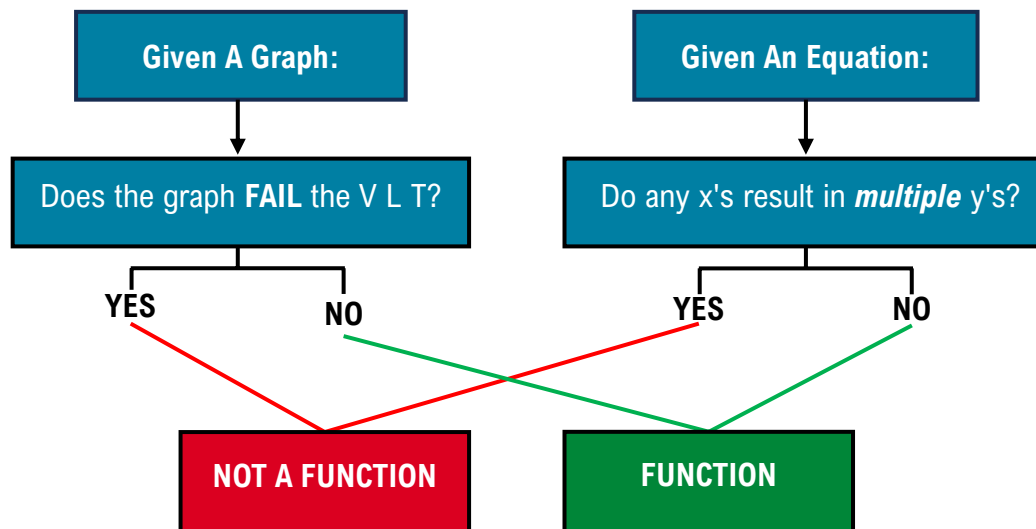


(C)



**TOPIC: INTRO TO FUNCTIONS & THEIR GRAPHS****Verifying if Equations are Functions**

- To verify if an **equation** is a function, you should _____ for _____ as a first step.



EXAMPLE: Determine if each of the equations below is **a function**, or **not a function**.

(A)

$$y + 4 = 3x$$

x		y
-1		
0		
1		
2		

If equation is a _____, then it is a

[**FUNCTION** | **NOT A FUNCTION**].

(B)

$$x^2 + y^2 = 25$$

x		y
-1		
0		
1		
2		

If equation has an _____ power of y , then it is a

[**FUNCTION** | **NOT A FUNCTION**].

- If an equation is a function, we can write it using function _____ (replace _____ with _____).

- We can write $y = 3x - 4$ as _____ = $3x - 4$



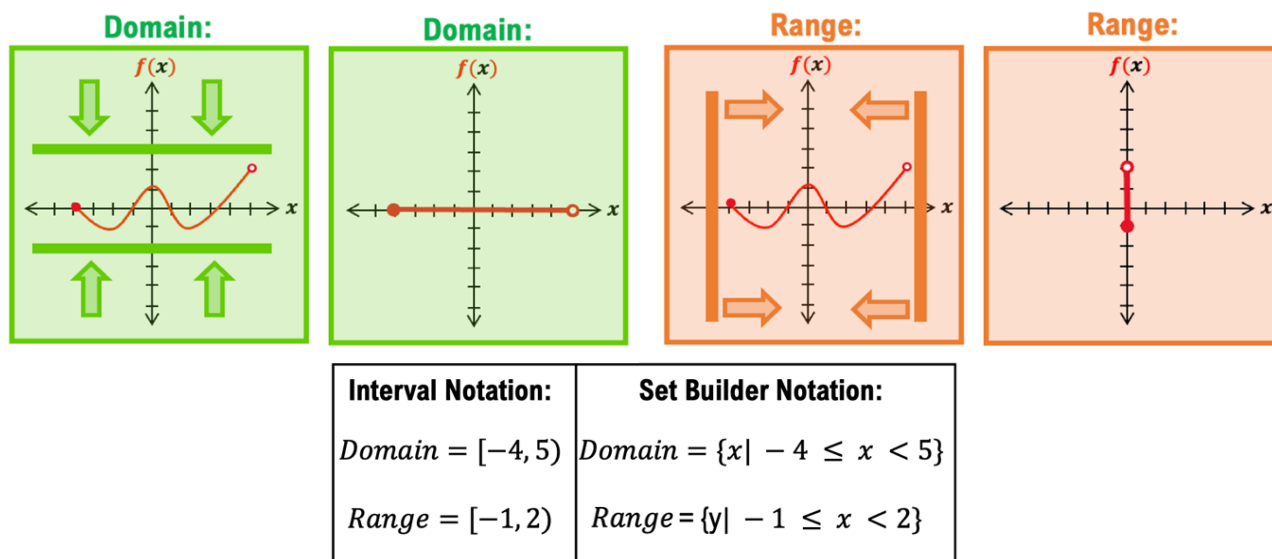
TOPIC: INTRO TO FUNCTIONS & THEIR GRAPHS

PRACTICE: Is the equation $y = -2x + 10$ a function? If so, rewrite it in function notation and evaluate at $f(3)$.

PRACTICE: Is the equation $y^2 + 2x = 10$ a function? If so, rewrite it in function notation and evaluate at $f(-1)$.

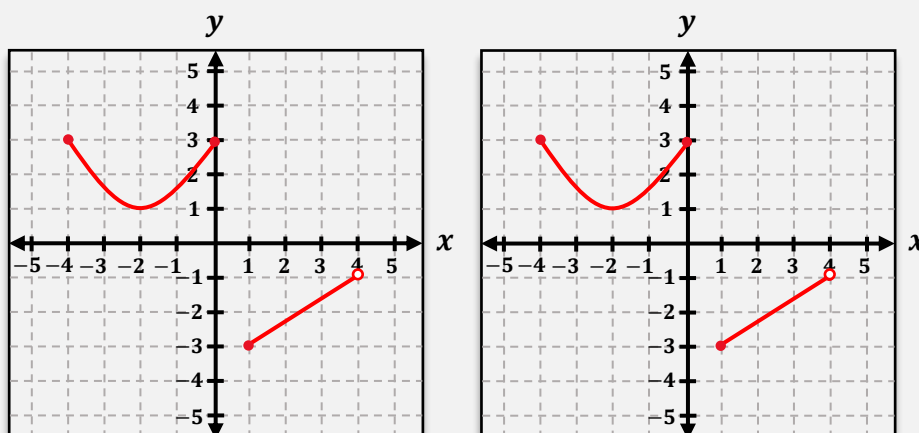
TOPIC: INTRO TO FUNCTIONS & THEIR GRAPHS**Finding The Domain And Range Of A Graph**

- The **domain** of a graph is the allowed ___-values, and the **range** of a graph is the allowed ___-values.
 - To find the **domain** of a graph, “**squish**” to the ___-axis. To find the **range**, “**squish**” to the ___-axis.



- The $[,], \leq, \geq$ symbols mean we **[INCLUDE | DON'T INCLUDE]** the value.
- The $(,), <, >$ symbols mean we **[INCLUDE | DON'T INCLUDE]** the value.

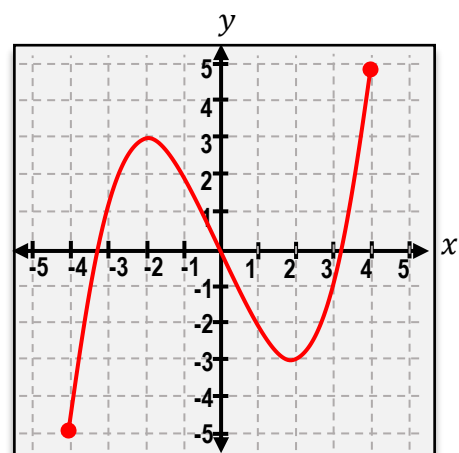
EXAMPLE: Determine the **domain** & **range** of the following graph below. Express the answer using interval notation.



- When we have multiple intervals or “jumps” in the graph, use the union symbol ($\cup$).

TOPIC: INTRO TO FUNCTIONS & THEIR GRAPHS

PRACTICE: Find the domain and range of the following graph (write your answer using interval notation).



**TOPIC: INTRO TO FUNCTIONS & THEIR GRAPHS****Finding the Domain of an Equation**

- You may need to find the **domain** (allowed x -values) of a function when given an equation instead of a graph.
 - First determine **restrictions** by identifying values that _____ the function. Two common situations:

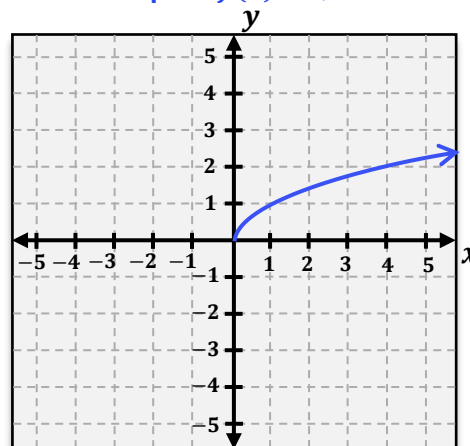
1) x inside of a Square Root

Domain: x -values that don't make the inside of the square root _____.

EXAMPLE: Find the domain of the function $f(x) = \sqrt{x}$ without graphing. Express using interval notation.

Restriction: x -values that make $\sqrt{\quad}$ negative: _____, therefore

Domain:

Graph of $f(x) = \sqrt{x}$:**2) x in the Denominator of a Fraction**

Domain: x -values that don't make the denominator _____.

EXAMPLE: Find the domain of the function $f(x) = \frac{2}{x-5}$ using interval notation.

Restriction: x -values that make denominator 0: _____, therefore

Domain:

**TOPIC: INTRO TO FUNCTIONS & THEIR GRAPHS**

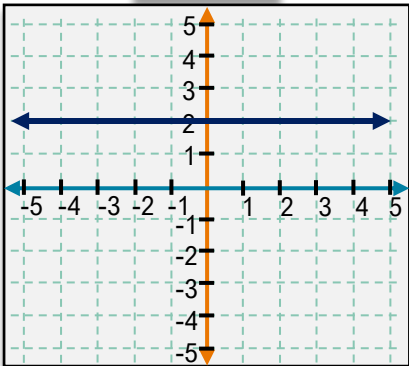
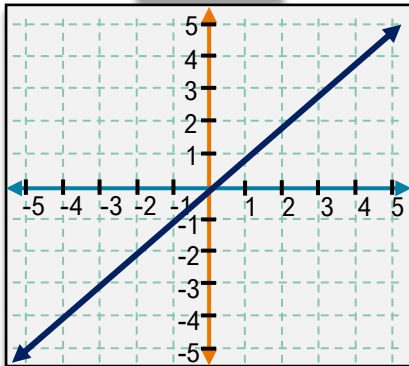
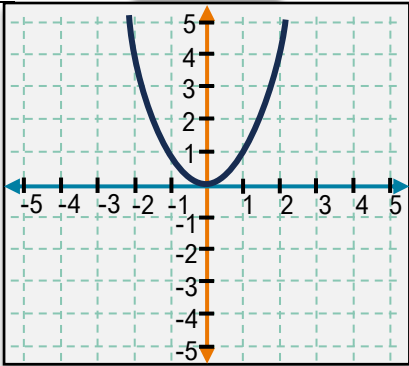
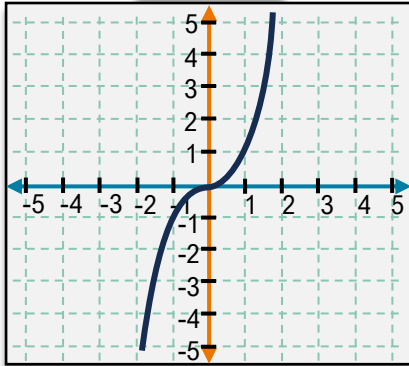
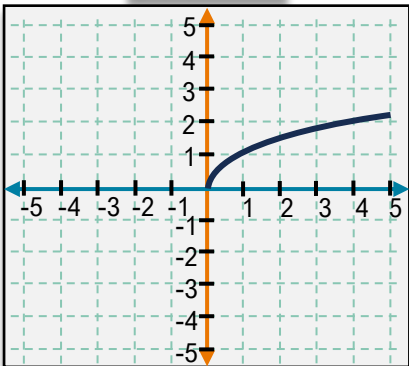
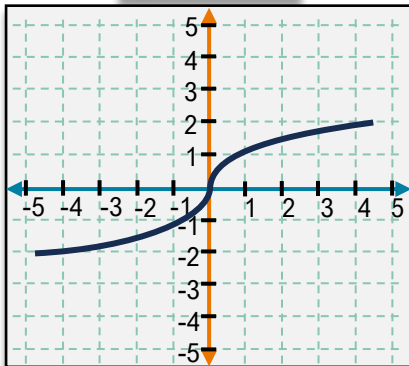
PRACTICE: Find the domain of $f(x) = \sqrt{x+4}$. Express your answer using interval notation.

PRACTICE: Find the domain of $f(x) = \frac{1}{x^2-5x+6}$. Express your answer using interval notation.

TOPIC: COMMON FUNCTIONS

Graphs of Common Functions

- There are several graphs that may _____ show up in this course.

Graphs of Common Functions	
Constant Function ($y = \text{same value, does NOT depend on } x$) $f(x) = c$ Domain: _____ Range: _____ 	Identity Function $f(x) = x$ Domain: _____ Range: _____ 
Square Function (Output = square of input) • Shape is a _____ $f(x) = x^2$ Domain: _____ Range: _____ 	Cube Function (Output = cube of input) $f(x) = x^3$ Domain: _____ Range: _____ 
Square Root Function (Output is the square root of input) • x [CAN CANT] be negative $f(x) = \sqrt{x}$ Domain: _____ Range: _____ 	Cube Root Function (Output is the cube root of input) • x [CAN CANT] be negative $f(x) = \sqrt[3]{x}$ Domain: _____ Range: _____ 

TOPIC: TRANSFORMATIONS

Intro to Transformations

- Transformations occur when a function is _____ and changes position and/or shape.

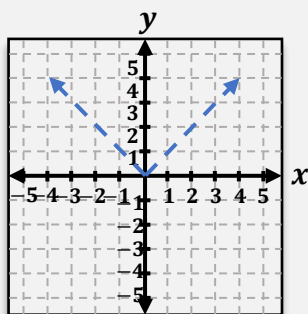
Transformations of Functions		
1) Reflection (FOLDING a function)	2) Shift (MOVING a function)	3) Stretch (SQUEEZING a function)
 $-f(x)$	 $f(x-h)+k$	 $c \cdot f(x)$

EXAMPLE: The graph of the $f(x) = |x|$ is shown below. Match the following functions $p(x)$, $q(x)$, & $r(x)$ to their corresponding graphs.

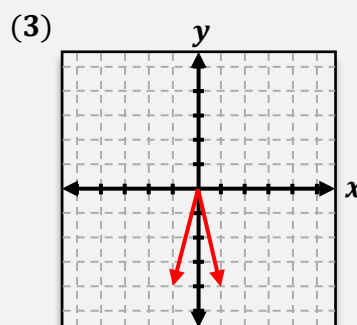
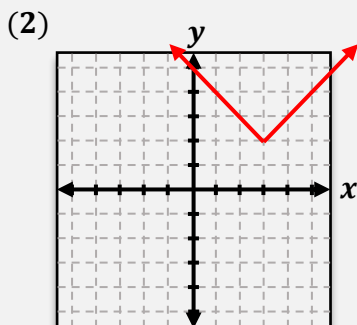
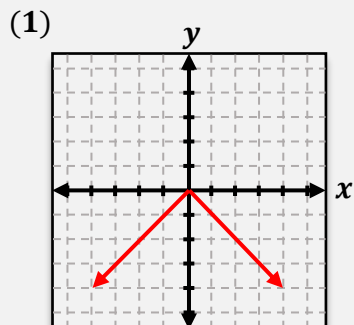
(A) $p(x) = |x - 3| + 2$

(B) $q(x) = -|x|$

(C) $r(x) = -4|x|$



Reflection	Shift	Stretch
$-f(x)$	$f(x-h)+k$	$c \cdot f(x)$



TOPIC: TRANSFORMATIONS

Reflections

Transformations of Functions

Reflection

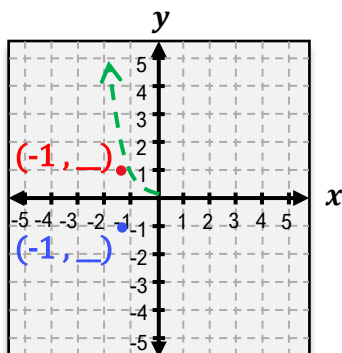
Shift

Stretch

- A reflection is a transformation where the function appears to be “_____” over the ____ -axis, or the ____ -axis.

REFLECTION OVER [X-AXIS | Y-AXIS]

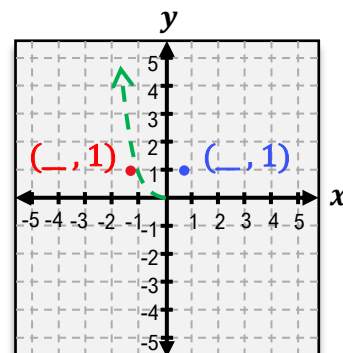
[x's | y's] change signs



$$f(x) \rightarrow -f(x)$$

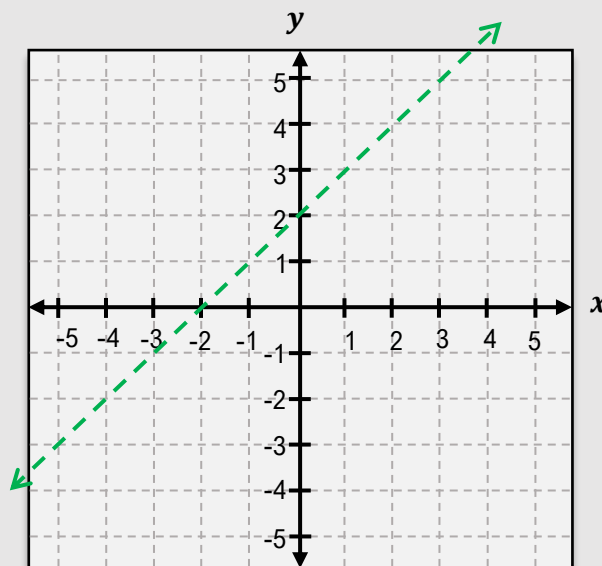
REFLECTION OVER [X-AXIS | Y-AXIS]

[x's | y's] change signs



$$f(x) \rightarrow f(-x)$$

EXAMPLE: Given $f(x) = x + 2$, let $h(x)$ be the function $f(x)$ with a **reflection** over the **x-axis**. Sketch the graph of $h(x)$, and determine which of the functions below matches the new function $h(x)$.



(A)

$$h(x) = x - 2$$

(B)

$$h(x) = -x + 2$$

(C)

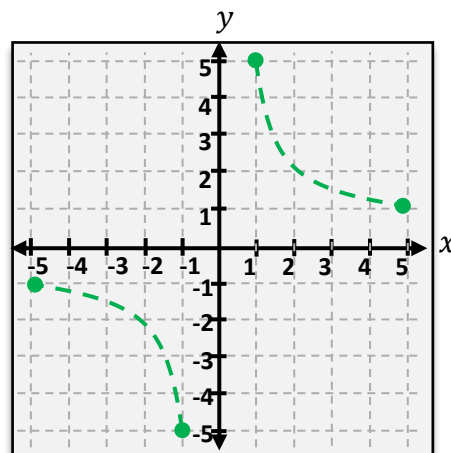
$$h(x) = -x - 2$$

(D)

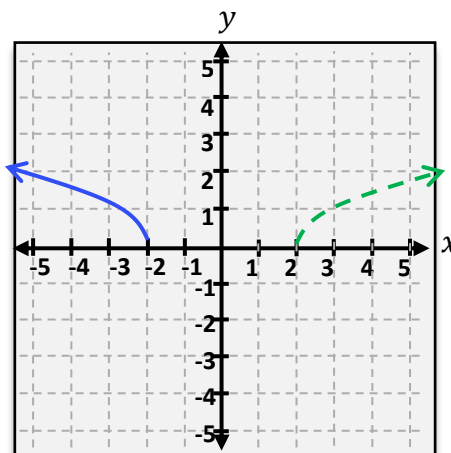
$$h(x) = x + 2$$

**TOPIC: TRANSFORMATIONS**

PROBLEM: Written below is a graph of the function $f(x)$. Sketch a graph of the function $g(x)$, where $g(x)$ is the function $f(x)$ after it has been reflected over the x -axis.



PRACTICE: Written below is a graph of the function $f(x) = \sqrt{x-2}$. If $g(x)$ is a reflection of $f(x)$ about the y -axis what is the equation for $g(x)$?



TOPIC: TRANSFORMATIONS

Shifting A Function

Transformations of Functions

Reflection

Shift

Stretch

- A shift occurs when a function is moved _____ and/or _____ from its original position.

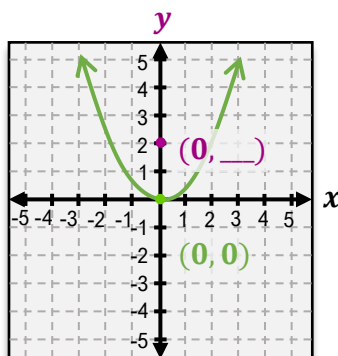
SHIFTS

General Form: $f(x) \rightarrow f(x - h) + k$

VERTICAL SHIFT ($h = 0$)

Vertical shifts change the
[x 's | y 's]

$$f(x) \rightarrow f(x - h) + k$$



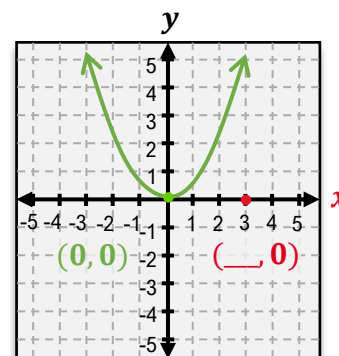
For $f(x) + k$, graph shifts [UP | DOWN]

For $f(x) - k$, graph shifts [UP | DOWN]

HORIZONTAL SHIFT ($k = 0$)

Horizontal shifts change the
[x 's | y 's]

$$f(x) \rightarrow f(x - h) + k$$



For $f(x + h)$, graph shifts [RIGHT | LEFT]

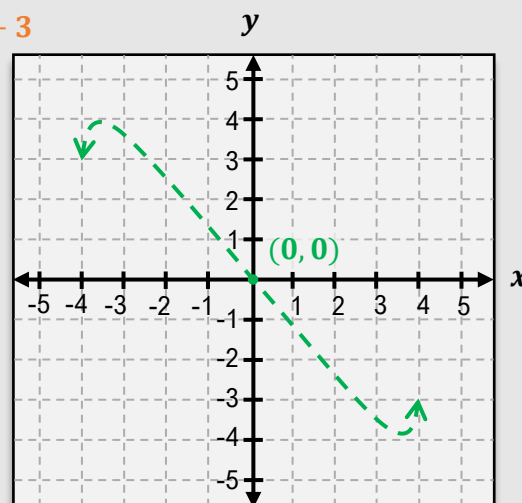
For $f(x - h)$, graph shifts [RIGHT | LEFT]

EXAMPLE: Given the function $f(x)$, sketch the transformation: $f(x - 2) + 3$

$$f(x) \rightarrow f(x - h) + k$$

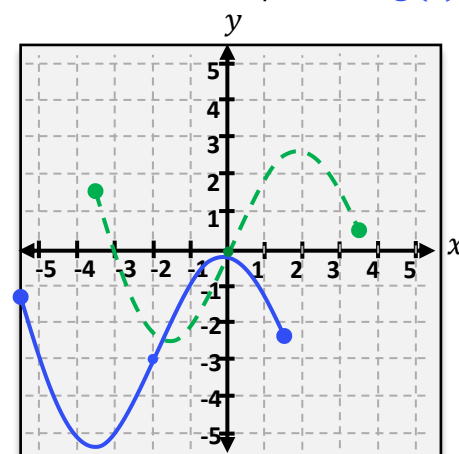
$h =$ _____

$k =$ _____



**TOPIC: TRANSFORMATIONS**

PRACTICE: The **green dotted line** in the graph below represents the function $f(x)$. The **blue solid line** represents the function $g(x)$, which is the function $f(x)$ after it has gone through a shift transformation. Find the equation for $g(x)$.



TOPIC: TRANSFORMATIONS

Graphs Of Shifted & Reflected Functions

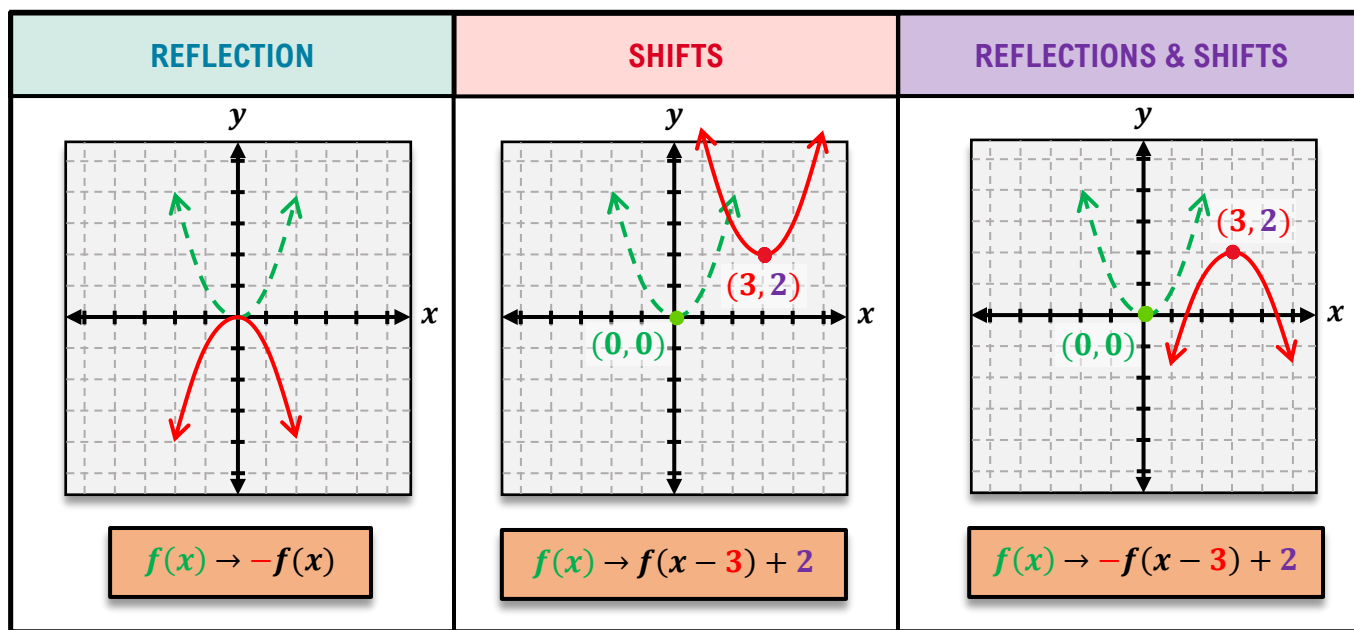
Transformations of Functions

Reflection

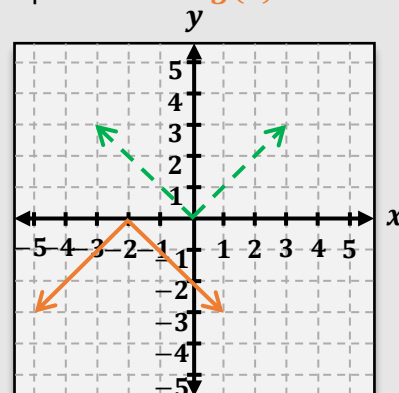
Shift

Stretch

- It is possible to see a _____ of transformations to a single function.

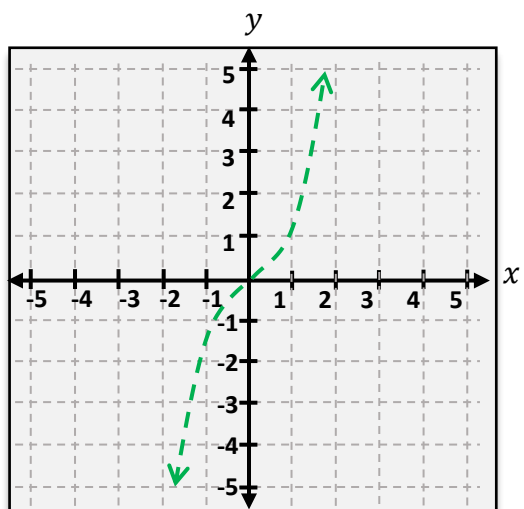


EXAMPLE: If $g(x)$ is a transformation of the function $f(x) = |x|$, write the equation for the $g(x)$.



**TOPIC: TRANSFORMATIONS**

PROBLEM: If $h(x)$ is a transformation of $f(x) = x^3$ where $f(x)$ is reflected over the x -axis, then shifted down 2 units. (A) write the equation for $h(x)$. (B) Sketch a graph of $h(x)$.





TOPIC: TRANSFORMATIONS

Stretches And Shrinks

Transformations of Functions

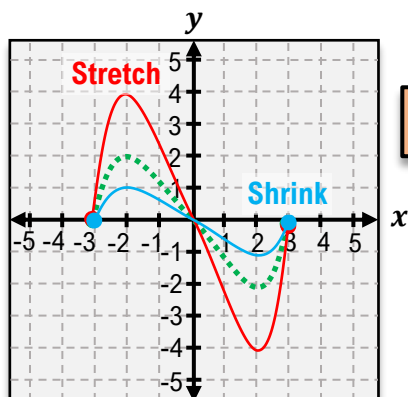
Reflection

Shift

Stretch

- Stretches/Shrinks occur when a _____ (c) is multiplied inside or outside the function.

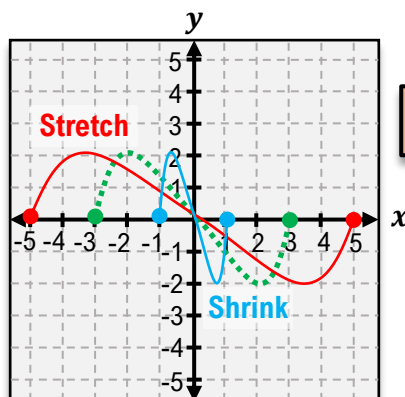
Vertical Stretch/Compress



$$f(x) \rightarrow c \cdot f(x)$$

- When $0 < c < 1$ the graph vertically [**STRETCHES** | **SHRINKS**]
- When $c > 1$ the graph vertically [**STRETCHES** | **SHRINKS**]

Horizontal Stretch/Compress



$$f(x) \rightarrow f(c \cdot x)$$

- When $0 < c < 1$ the graph horizontally [**STRETCHES** | **SHRINKS**]
- When $c > 1$ the graph horizontally [**STRETCHES** | **SHRINKS**]

EXAMPLE: Graphed below is the function $f(x)$. Sketch the graphs of the following functions:

(A)

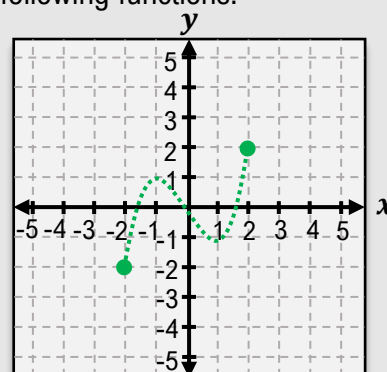
$$2 \cdot f(x)$$

(B)

$$\frac{1}{2} \cdot f(x)$$

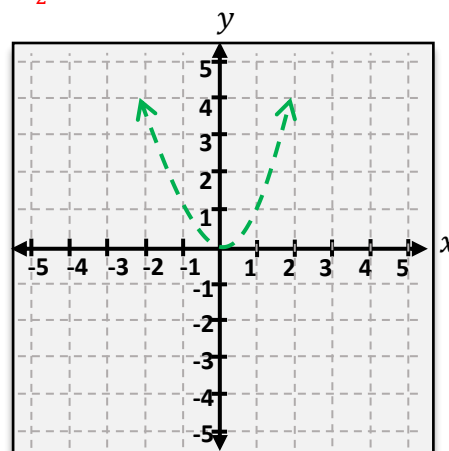
(C)

$$f\left(\frac{1}{2} \cdot x\right)$$



**TOPIC: TRANSFORMATIONS**

PROBLEM: Given that $f(x) = (c \cdot x)^2$, graph $f(x)$ when (A) $c = 2$, (B) $c = \frac{1}{2}$



**TOPIC: TRANSFORMATIONS****Domain & Range of Transformed Functions**

- A transformation can _____ the domain and range of a function.
 - We can find the domain & range of a transformed function by _____ the new graph.

Function: $f(x)$	Function: $f(x - 1) + 2$
Domain $\rightarrow D: [-3, 3]$ Range $\rightarrow R: [-3, 3]$	Domain $\rightarrow D: [__, __]$ Range $\rightarrow R: [__, __]$

EXAMPLE: Given the function $f(x) = x^2$, sketch a graph of the function $g(x) = (x - 3)^2 + 2$ and determine its domain and range.

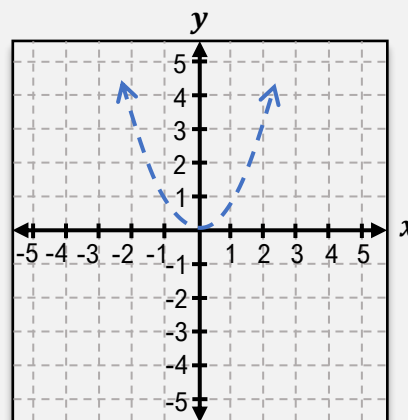
$$f(x) \rightarrow f(x - h) + k$$

Domain Of $f(x)$: $(-\infty, \infty)$

Range Of $f(x)$: $[0, \infty)$

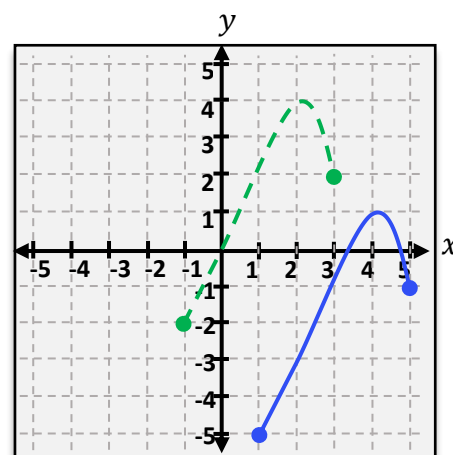
Domain Of $g(x)$:

Range Of $g(x)$:



TOPIC: TRANSFORMATIONS

PRACTICE: The **green dotted curve** below is a graph of the function $f(x)$. Find the domain and range of $g(x)$ (the **blue solid curve**), which is a transformation of $f(x)$.



Domain of $f(x) = [-1, 3]$

Range of $f(x) = [-2, 4]$

**TOPIC: FUNCTION COMPOSITION****Function Composition**

- Function composition is like evaluating, but you replace the inside variable of a function with **ANOTHER** _____.

EVALUATING a Function	COMPOSING a Function
$f(x) = x^2 + 3x - 10$ $f(7) = ()^2 + 3() - 10$ $=$ Result is a [NUMBER FUNCTION]	$f(x) = x^2 + 3x - 10$ $g(x) = x - 2$ $f(g(x)) = ()^2 + 3() - 10$ $=$ Result is a [NUMBER FUNCTION]

Note: $f(g(x))$ is often written as _____. First letter = outside function, second letter = inside function

EXAMPLE: Given the functions $f(x) = x + 4$ and $g(x) = x^2 - 3$, find the following composite functions (fully simplify your answer).

(A) $f(g(x)) =$

(B) $g(f(x)) =$

**TOPIC: FUNCTION COMPOSITION**

PRACTICE: Given the functions $f(x) = \sqrt{x+4}$ and $g(x) = (x-2)^2 - 4$, (A) find $(f \circ g)(x)$ and (B) $(g \circ f)(x)$.

PRACTICE: Given the functions $f(x) = \frac{1}{x^2-2}$ and $g(x) = \sqrt{x+2}$, (A) find $(f \circ g)(x)$ and (B) $(g \circ f)(x)$.



TOPIC: FUNCTION COMPOSITION

Evaluating Composed Functions

- You may have to compose functions and **then** _____ at a specific value, $f(g(\#))$. Two common methods:

Method 1: Compose $\rightarrow$ Evaluate <i>Use when first asked to find $f(g(x))$</i>	Method 2: Evaluate inside $\rightarrow$ Evaluate outside
Diagram illustrating Method 1: Compose $\rightarrow$ Evaluate. It shows $f(x)$ and $g(x)$ with arrows pointing to $f(g(x))$, which then has an arrow pointing to $f(g(\#))$.	Diagram illustrating Method 2: Evaluate inside $\rightarrow$ Evaluate outside. It shows $f(x)$ with a curved arrow pointing to $f(g(\#))$, and $g(x)$ with an arrow pointing to $g(\#)$, which then has an arrow pointing to $f(g(\#))$.
<u>EXAMPLE:</u> For $f(x) = x^2$ and $g(x) = x - 1$, find $f(g(x))$ and then evaluate $f(g(3))$	<u>EXAMPLE:</u> For $f(x) = x^2$ and $g(x) = x - 1$, evaluate $f(g(3))$

PRACTICE: Given the functions $f(x) = x + 3$ and $g(x) = x^2$, (A) find $(f \circ g)(2)$ and (B) $(g \circ f)(2)$.

**TOPIC: FUNCTION COMPOSITION****Domain Of Composite Functions**

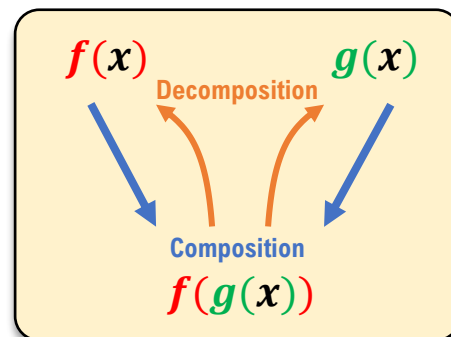
- To find the **domain** of a composite function, follow these steps to exclude x -values from $f \circ g(x)$:
 - 1) Find any x -values not defined for _____
 - 2) Find any x -values that make $g(x)$ not defined for _____

EXAMPLE: Given the functions $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$, determine $f \circ g(x)$ and its domain.

PRACTICE: Given the functions $f(x) = x^2$ and $g(x) = \sqrt{x-8}$ find $(f \circ g)(x)$ and determine its domain.

**TOPIC: FUNCTION COMPOSITION****Decomposing Functions**

- Function **decomposition** is the _____ of function **composition**.
 - There are many correct answers when decomposing functions.



EXAMPLE: Express the function $h(x) = \sqrt{x - 2}$ as a composition of two functions f & g so that $h(x) = (f \circ g)(x)$.

PROBLEM: Express the function $h(x) = \frac{1}{x^2 + 3x - 10}$ as a composition of two functions f & g so that $h(x) = (f \circ g)(x)$.