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TOPIC: SOLVING LINEAR EQUATIONS

● A **linear expression** with an ____ sign is a **linear equation**!

Linear Expression	Linear Equation
$2x + 3$	$2x + 3$ ______
When x is 4, $\underbrace{2(4) + 3}_{11}$	
● Simplify/evaluate for ***KNOWN*** x	● Solve for ***UNKNOWN*** x ▪ Find value(s) of x that make the statement ________

● You will need to use different operations ($+,\ -,\ \times,\ \div$) to __________ x.

▪ These operations should ***ALWAYS*** be done to _____________ of the equation.

EXAMPLE: Identify and perform the operation needed to **isolate x** by applying it to both sides.

$(\boldsymbol{A})$ $x + 2 = 0$

$(\boldsymbol{B})$ $3x = 12$

__________ **Operations**

$+$

$\times$

● You'll often have to do *multiple* operations to solve a linear equation.

EXAMPLE: Solve the equation.

$$2(x - 3) = 0$$

SOLVING LINEAR EQUATIONS	
Simplifying Algebraic Expressions	**1)** Distribute constants
	2) Combine like terms
	3) Group terms w/ __ & __________ on opposite sides
	4) Isolate x (Solve for x)
	5) _______ solution by replacing x in original equation

▪ ____ is the __________ or ________ of the equation

TOPIC: SOLVING LINEAR EQUATIONS

PRACTICE: Solve the equation.

$$3(2-5x)=4x+25$$

SOLVING LINEAR EQUATIONS
1) Distribute constants
2) Combine like terms
3) Group terms w/ $\boldsymbol{x}$ & constants on opposite sides
4) Isolate $\boldsymbol{x}$
5) Check solution by replacing $\boldsymbol{x}$ in original equation

Linear Equations with Fractions

- Linear equations may have **fractions** that need to be ____________ using **L**east **C**ommon **D**enominator first.

EXAMPLE: Solve the equation.

$$\frac{1}{4}(x+2)-\frac{1}{3}x=\frac{1}{12}$$

SOLVING LINEAR EQUATIONS
0) Multiply by ______ to eliminate fractions
1) Distribute constants
2) Combine like terms
3) Group terms w/ $\boldsymbol{x}$ & constants on opposite sides
4) Isolate $\boldsymbol{x}$
5) (Optional) Check by replacing $\boldsymbol{x}$ in original eqn

PRACTICE: Solve the equation.

$$\frac{9}{2}+\frac{1}{4}(x+2)=\frac{3}{4}x$$

TOPIC: SOLVING LINEAR EQUATIONS
Categorizing Linear Equations

● **Linear equations** can be put in 3 possible **categories** based on ____________ solutions they have.

- These solutions may be written as a ________________.

EXAMPLE: Solve, then categorize the linear equations.

______________	______________	______________
$2x + 4 = 10$ ● ________ **solution** ● Solution set is _______	$x + 5 = x + 2 + 3$ ● ________ statement ● __________ **solutions** ● Solution set is _______ (all real numbers)	$x = x + 4$ ● ________ statement ● _____ solutions ● Solution set is ___, _________ set

PRACTICE: Solve the equation. Then state whether it is an identity, conditional, or inconsistent equation.

$$5x + 17 = 8x + 12 - 3(x + 4)$$

SOLVING LINEAR EQUATIONS
0) Multiply by LCD to eliminate fractions
1) Distribute constants
2) Combine like terms
3) Group terms w/ x & constants on opposite sides
4) Isolate x (if any x terms remain)

TOPIC: SOLVING LINEAR EQUATIONS

PRACTICE: Solve the equation. Then state whether it is an identity, conditional, or inconsistent equation.

$$\frac{x}{4}+\frac{1}{6}=\frac{x}{3}$$

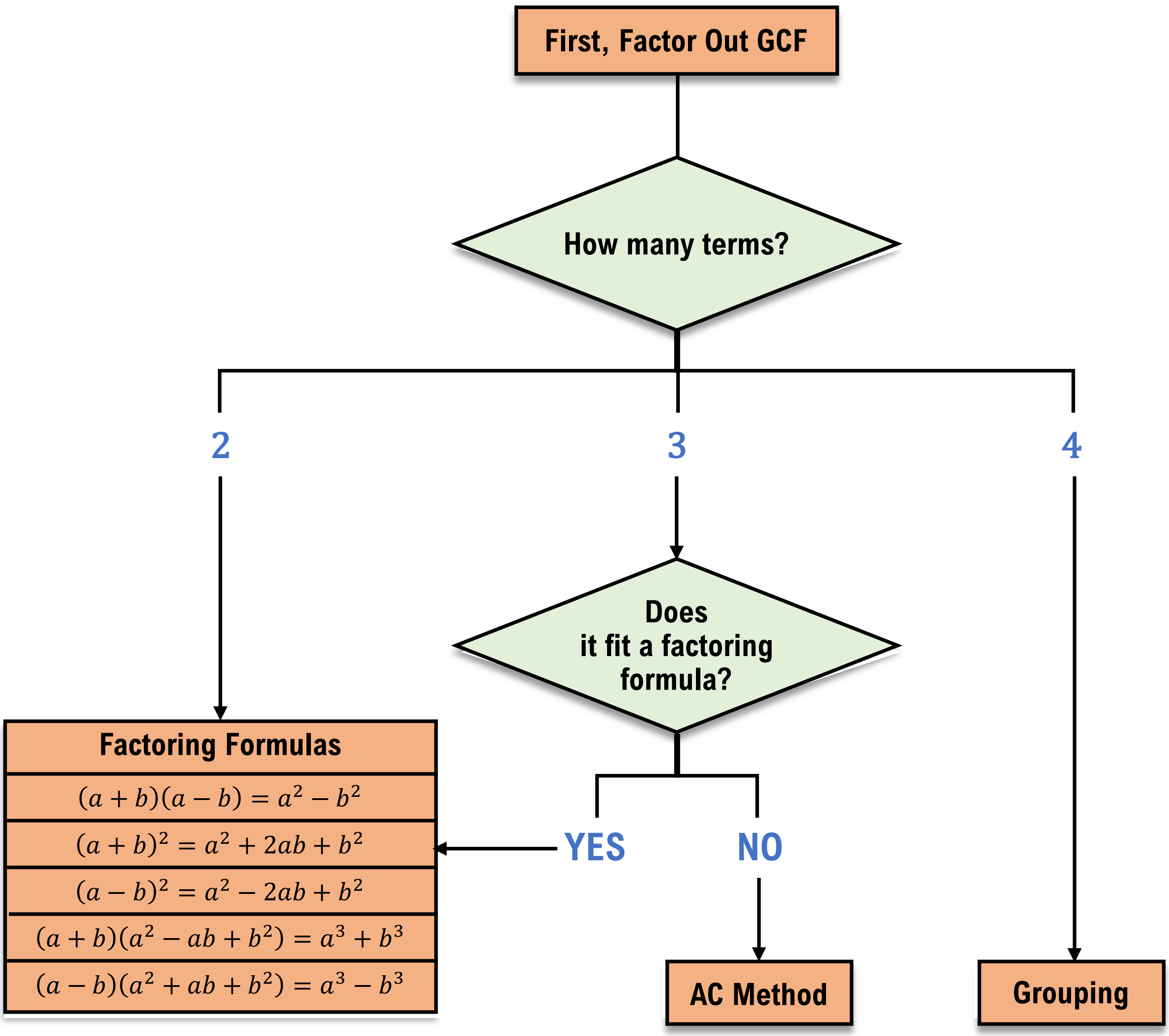
SOLVING LINEAR EQUATIONS
0) Multiply by LCD to eliminate fractions
1) Distribute constants
2) Combine like terms
3) Group terms w/ $\boldsymbol{x}$ & constants on opposite sides
4) Isolate $\boldsymbol{x}$ (if any $\boldsymbol{x}$ terms remain)

PRACTICE: Solve the equation. Then state whether it is an identity, conditional, or inconsistent equation.

$$-2(5-3x)+x=7x-10$$

TOPIC: INTRO TO QUADRATIC EQUATIONS

Topic Resource: Choosing a Factoring Method



Repeat this process with each non – prime factor.

TOPIC: INTRO TO QUADRATIC EQUATIONS

- If you take a **linear equation** and add an ____ term, you get a **quadratic equation!**
  - Also called a polynomial of *degree 2.*

$$\_\_\_\_\_\_\_ \; 2x - 6 = 0$$

- You will often need to write quadratic equations in *standard form.*

$$\boxed{ax^2 + bx + c = 0}$$

  - All terms are on the **same** side in *descending* order of __________.

EXAMPLE: Write each given quadratic equation in standard form. Identify $a, b,$ and c.

(A) $5x^2 = x - 3$

$a =$ ______

$b =$ ______

$c =$ ______

(B) $-2x^2 + \frac{5}{3} = 0$

$a =$ ______

$b =$ ______

$c =$ ______

PRACTICE**:** Write the given quadratic equation in standard form. Identify $a, b,$ and c.

$-4x^2 + x = 8$

$a =$ ______

$b =$ ______

$c =$ ______

TOPIC: INTRO TO QUADRATIC EQUATIONS

Factoring

- To **solve** a quadratic equation, we want to find every value of _____ that makes the equation __________.
  - There are often ____ correct values of x, which we call the ***solutions, roots,*** or ***zeros***.

Solving Linear Equations	Solving Quadratic Equations
$2x - 6 = 0$	$x^2 - 5x + 4 = 0$
$2x = 6$	
$x = 3$	
	• We will need to use something else to solve quadratic equations:

- One way to solve a quadratic equation is to **factor** from standard form, then set each factor equal to ___.

$$x^2 + x - 6 = 0$$
$$\underbrace{(x+3)}_{0}\underbrace{(x-2)}_{0} = 0$$

  - If the factors **multiplied** $= 0$, one of the **factors** must $= 0$ to make this true.

EXAMPLE: Solve the given quadratic equation by factoring.

$x^2 - 9x = -20$

Multiply to $a \cdot c$	Add to b

FACTORING QUAD. EQNS

1) Write eqn in ________ form

2) ________ completely

3) Set factors = __, solve for x

4) Check solutions in original eqn

TOPIC: INTRO TO QUADRATIC EQUATIONS

PRACTICE: Solve the given quadratic equation by factoring.

$$3x^2 + 12x = 0$$

FACTORING QUAD. EQNS
1) Write eqn in standard form
2) Factor completely
3) Set factors = 0, solve for x
4) Check solutions

PRACTICE: Solve the given quadratic equation by factoring.

$$2x^2 + 7x + 6 = 0$$

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TOPIC: The Square Root Property

- You won't always be able to **factor** to solve quadratics.
  - There are **3 other methods** that you can use:

$x^2 + x - 6 = 0$	$x^2 - 5 = 0$
$(x \qquad)(x \qquad) = 0$	$(x \qquad)(x \qquad) = 0$
Factorable? [YES | NO]	Factorable? [YES | NO]

<table>
<tr><th colspan="5">SOLVING QUADRATIC EQUATIONS</th></tr>
<tr><td colspan="5">$ax^2 + bx + c = 0$
(Standard form)</td></tr>
<tr><td></td><td>FACTORING</td><td>SQUARE ROOT PROPERTY</td><td>METHOD #3</td><td>METHOD #4</td></tr>
<tr><td>USE IF</td><td>● Has ________ factors OR
● No _______ term ($c =$ __)</td><td>● $(x + \#)^2 = [\textbf{constant}]$ OR
● No __ term ($b =$ __)</td><td></td><td></td></tr>
<tr><td>STEPS</td><td>1) Write eqn in standard form
2) Factor completely
3) Set factors = 0, solve for x
4) Check solutions</td><td>1) _______ squared expression
2) Take __ & __ square root
3) Solve for x
4) (Optional) Check solutions</td><td></td><td></td></tr>
</table>

EXAMPLE: Solve the given quadratic equations using the square root property.

(A) $(x + 1)^2 = 4$

(B) $4x^2 - 5 = 0$

- Solutions aren't always whole numbers! They could have ___________ and/or ____________.

PRACTICE: Solve the given quadratic equation using the square root property.

$$\left(x - \frac{1}{2}\right)^2 - 5 = 0$$

TOPIC: The Square Root Property

PRACTICE: Solve the given quadratic equation using the square root property.

$$2x^2 - 16 = 0$$

Imaginary Roots

- You may get imaginary (or complex) roots when using the square root property.
  - Simplify them as you would any complex number!

EXAMPLE: Solve the given quadratic equation using the square root property.

$$4x^2 + 25 = 0$$

SQ. ROOT PROPERTY
1) Isolate squared expression
2) Take $+$ & $-$ square root
3) Solve for x
4) (Optional) Check solutions

Note: When a and c have the same sign in standard form, you will *always* end up with a complex answer.

TOPIC: COMPLETING THE SQUARE

● If a quadratic isn't in the form $(x + \#)^2 = [\textbf{constant}]$, you can *put it in that form* by **completing the square**.

$(x+3)^2 = 2$

↓ Square Root Property

$\sqrt{(x+3)^2} = \sqrt{2}$

$x^2 + 6x + \_\_\_\_ = -7 + \_\_\_\_$

↓ Complete the Square

$(x + \#)^2 = [\textbf{constant}]$

↓ Square Root Property

$\sqrt{(x+\#)^2} = \sqrt{[\textbf{constant}]}$

SOLVING QUADRATIC EQUATIONS				
$ax^2 + bx + c = 0$ *(Standard form)*				
	FACTORING	**SQ. ROOT PROPERTY**	**COMPLETE THE SQUARE**	**METHOD #4**
USE IF	● Has obvious factors **OR** ● $c = 0$	● $(x + \#)^2 = [\textbf{constant}]$ **OR** ● $b = 0$	● Leading coeff. (a) is ___ **AND** ● b is _______	
STEPS	**1)** Write eqn in standard form **2)** Factor completely **3)** Set factors = 0, solve for x **4)** Check solutions	**1)** Isolate squared expression **2)** Take + & − square root **3)** Solve for x **4)** Check solutions	**1)** Simplify eqn to $x^2 + bx = c$ **2)** Add ___ to both sides **3)** Factor to $(x + \_\_\_)^2$ **4)** Solve using __________ prop.	

EXAMPLE: Solve the quadratic equation by completing the square.

(A)

$x^2 + 6x \quad\quad = -7$

$(x + \_\_\_)^2 =$

(B)

$x^2 + 8x + 1 = 0$

$(x + \_\_\_)^2 =$

$a^2 + 2ab + b^2 = (a + b)^2$

(Perfect Square Trinomial)

$x^2 + bx + \_\_\_\_\_ = c + \_\_\_\_\_\_$

$(x + \_\_\_)^2 = [\textbf{constant}]$

TOPIC: COMPLETING THE SQUARE

PRACTICE**:** Solve the given quadratic equation by completing the square.

$x^2 + 3x - 5 = 0$

COMPLETE THE SQUARE
1) Simplify eqn to $\boldsymbol{x^2 + bx = c}$
2) $+\left(\frac{b}{2}\right)^2$ to both sides
3) Factor to $\left(x + \frac{b}{2}\right)^2$
4) Solve using sqrt. prop.

PRACTICE**:** Solve the given quadratic equation by completing the square.

$3x^2 - 6x - 9 = 0$

TOPIC: THE QUADRATIC FORMULA

- The solutions to *ANY* quadratic equation in standard form can be found using the **quadratic formula.**

$$x = \underline{\hspace{4cm}}$$

(Quadratic Formula)

	SOLVING QUADRATIC EQUATIONS			
	$ax^2 + bx + c = 0$ *(Standard form)*			
	FACTORING	**SQ. ROOT PROPERTY**	**COMPLETE THE SQUARE**	**QUADRATIC FORMULA**
USE IF	• Has obvious factors **OR** • $c = 0$	• $(x + \#)^2 = [\textbf{constant}]$ **OR** • $b = 0$	• Leading coeff. is 1 **AND** • b is even	• Can't easily _______ • Unsure what method to use
STEPS	**1)** Write eqn in standard form **2)** Factor completely **3)** Set factors = 0, solve for x **4)** Check solutions	**1)** Isolate squared expression **2)** Take + & – square root **3)** Solve for x **4)** Check solutions	**1)** Simplify eqn to $\boldsymbol{x^2 + bx = c}$ **2)** + $\left(\frac{b}{2}\right)^2$ to both sides **3)** Factor to $\left(x + \frac{b}{2}\right)^2$ **4)** Solve using sqrt prop.	**1)** Write eqn in standard form **2)** Plug a, b, c in quad. form. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **3)** Compute & _______ sol'ns

EXAMPLE: Solve each equation using the quadratic formula.

(A)

$$x^2 + 2x - 3 = 0$$

$$\boldsymbol{x} = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

$\boldsymbol{x} = \_\_\_\_\_$ & $\boldsymbol{x} = \_\_\_\_\_$

(B)

$$x^2 - 5x = -1$$

$$\boldsymbol{x} = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

$\boldsymbol{x} = \_\_\_\_\_$ & $\boldsymbol{x} = \_\_\_\_\_$

TOPIC: THE QUADRATIC FORMULA

QUADRATIC FORMULA
1) Write eqn in standard form
2) Plug a, b, c in quad. form. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3) Compute & simplify sol'ns

PRACTICE**:** Solve the given quadratic equation using the quadratic formula.

$$3x^2 + 4x + 1 = 0$$

PRACTICE**:** Solve the given quadratic equation using the quadratic formula.

$$2x^2 - 3x = -3$$

TOPIC: THE QUADRATIC FORMULA
The Discriminant

- You can find **how many real** or **imaginary** solutions a quadratic has *WITHOUT* solving by using the discriminant.
  - **Discriminant:** the expression under the radical in the quadratic formula.
  - ________ determines number/type of solutions: Discriminant is **Positive** = ____ real solution(s)

Zero = ____ real solution(s)

Negative = ____ real solution(s)

____ __________ solutions

$$x = \frac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$$

(Quadratic Formula)

(Discriminant)

EXAMPLE: For the following quadratic equations, determine the number and type of solutions. Do not solve.

(A)	**(B)**	**(C)**
$2x^2 + 3x - 2 = 0$	$4x^2 + x + 2 = 0$	$x^2 - 10x + 25 = 0$
$b^2 - 4ac$ is: [$+$ | 0 | $-$]	$b^2 - 4ac$ is: [$+$ | 0 | $-$]	$b^2 - 4ac$ is: [$+$ | 0 | $-$]

PRACTICE: Determine the number and type of solutions of the given quadratic equation. Do not solve.

$x^2 + 8x + 16 = 0$

PRACTICE: Determine the number and type of solutions of the given quadratic equation. Do not solve.

$-4x^2 + 4x + 5 = 0$

TOPIC: COMPLEX NUMBERS
Introduction to Complex Numbers

● **Recall:** We've learned *real & imaginary numbers* separately, but you'll see expressions with *both* types of numbers.

- We call these **complex numbers**, which have a **standard form** of:

$$\boxed{a + b\boldsymbol{i}}$$

a is the ________ part ***b*** is the ____________ part

EXAMPLE: Identify the real and imaginary parts of each complex number.

COMPLEX NUMBERS

$(\boldsymbol{A})$ $4 - 3i$

$a =$ _____ $b =$ _____

$(\boldsymbol{B})$ $0 + 7i$

$a =$ _____ $b =$ _____

$(\boldsymbol{C})$ $2 + 0i$

$a =$ _____ $b =$ _____

PRACTICE: Identify the real and imaginary parts of the complex number.

$-4 - 9i$

$a =$ _____ $b =$ _____

PRACTICE: Identify the real and imaginary parts of the complex number.

$3 + 2i\sqrt{3}$

$a =$ _____ $b =$ _____

PRACTICE: Write the complex number in standard form.

$\frac{9+\sqrt{-16}}{3}$

TOPIC: COMPLEX NUMBERS
Adding & Subtracting Complex Numbers

- Just like with algebraic expressions, when you add or subtract complex #s, simply **combine like terms.**
  - Always express your answer in ____________ form!

Adding/Subtracting Algebraic Expressions

$$(4+8x)+(2+3x)$$

$$4+8x+2+3x$$

$$6 \quad + \quad 11x$$

EXAMPLE: Perform the given operation, expressing the result in standard form.

ADDING COMPLEX NUMBERS	SUBTRACTING COMPLEX NUMBERS
$(4+8i)+(2+3i)$	$(4+8i)-(2+3i)$

PRACTICE: Find the difference. Express your answer in standard form.

$$(2+8i)-(4-i)$$

PRACTICE: Find the sum. Express your answer in standard form.

$$5(4+7i)+6(3-2i)$$

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TOPIC: COMPLEX NUMBERS
Multiplying Complex Numbers

- Complex numbers are multiplied just like algebraic expressions! We A) ______________ or B) ________
  - Multiplying will *ALWAYS* produce an $\boldsymbol{i^2}$ term that will get simplified.

EXAMPLE: Find the product. Write answers in standard form.

$(\boldsymbol{A})$

$3i(7-2i)$

$(\boldsymbol{B})$

$(-6+2i)(3+4i)$

MULTIPLYING COMPLEX NUMBERS
1) Distribute or FOIL
2) Apply $\boldsymbol{i^2}=-1$
3) Combine like terms

PRACTICE: Perform the indicated operation. Express your answer in standard form.

$(3+8i)^2$

PRACTICE: Find the product. Express your answer in standard form.

$2i(9-4i)(6+5i)$

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TOPIC: COMPLEX NUMBERS
Complex Conjugates

● Reverse the ______ of *only* the **imaginary** part of a complex number to get the **conjugate:** $a + b\boldsymbol{i} \Leftrightarrow$

EXAMPLE: Find the conjugate of each complex number.

(A) $1 + 2i$ **(B)** $1 - 2i$ **(C)** $-1 + 2i$

● Multiplying **complex conjugates** (by FOIL) ***ALWAYS*** results in a ________ number

EXAMPLE: Find the product.

$$(2 + 3i)(2 - 3i)$$

$(a + bi)(a - bi) =$ ____________

PRACTICE: Find the product of the given complex number and its conjugate.

$$4 - 5i$$

PRACTICE: Find the product of the given complex number and its conjugate.

$$-7 - i$$

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TOPIC: COMPLEX NUMBERS
Dividing Complex Numbers

- Dividing by a complex number results in a fraction with $\boldsymbol{i}$ in the bottom: this is **BAD**
  - Denominators should ***ALWAYS*** be real! To do this, multiply by its ____________

$$\frac{c}{a + \cancel{bi}}$$

EXAMPLE: Find the quotient. Write answer in standard form.

$$\frac{3}{1 + 2i}$$

DIVIDING COMPLEX NUMBERS

1) Multiply top AND bottom by complex conj. of bottom & simplify

2) Expand fraction into real & imaginary parts

3) Simplify fractions to lowest terms

PRACTICE: Find the quotient. Express your answer in standard form.

$$\frac{6 + i}{4 - 2i}$$

PRACTICE: Find the quotient. Express your answer in standard form.

$$\frac{-5 + 3i}{-7 - 4i}$$

TOPIC: SOLVING RATIONAL EQUATIONS

- A **rational equation** is an equation with a __________ in the ________________ of a fraction.
  - We can solve a rational equation by turning it into a ____________ equation.

Rational Equation

$$\frac{1}{x-1} = 12$$

- Solutions ***CANNOT*** be any value that makes a denominator ______; this is the ______________.

EXAMPLE: Solve the rational equation.

$$\frac{x}{x-1} = \frac{7}{6} \qquad x \neq \_\_\_\_$$

SOLVING RATIONAL EQUATIONS
1) Determine __________ by setting denom. = ____
2) Multiply by ____ to eliminate fractions
3) Solve linear equation
4) Check solution with restriction

TOPIC: SOLVING RATIONAL EQUATIONS

Solution Equal to Restriction

EXAMPLE: Solve the rational equation.

$$\frac{x-5}{x-2} = \frac{-3}{x-2} + 6 \qquad x \neq \_\_\_\_$$

SOLVING RATIONAL EQUATIONS
1) Determine restriction by setting denom. $=\ 0$
2) Multiply by LCD to eliminate fractions
3) Solve linear equation
4) Check solution with restriction

● If your answer is **equal** to the **restriction**, then there is ___________________, i.e. solution set = _____.

PRACTICE: Solve the equation.

$$\frac{2x+4}{x-1} = 5$$

PRACTICE: Solve the equation.

$$\frac{5}{x} - \frac{2}{3x} = 4 + \frac{3}{x}$$

PRACTICE: Solve the equation.

$$\frac{-5}{x+4} - 3 = \frac{x-1}{x+4}$$

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TOPIC: LINEAR INEQUALITIES
Interval Notation

$\{x \mid -13 < x \leq 9\}$

$(-13\,,9]$

- Instead of set notation, a more compact way to express solution sets is in **interval notation**.

	CLOSED INTERVAL		OPEN INTERVAL		HALF CLOSED/ OPEN INTERVAL
	endpoint				
Set Notation	$\{x \mid 0 \leq x \leq 5\}$	$\leq$	$\{x \mid 0 < x < 5\}$	$<$	$\{x \mid 0 \leq x < 5\}$
Interval Notation	[___ , ___]	[	(___ , ___)	(	______
Number Line		●		o	
	• Endpoints ___cluded		• Endpoints ___cluded		

EXAMPLE: Express the set in interval notation & graph on a number line.

$\{x \mid x \geq 3\}$

______

- When solution sets have no "end point," we use infinity symbols ______ and _______.
  - In interval notation, *always* express ∞ with _________ bounds.

PRACTICE: Express the given set in interval notation and graph.

$\{x \mid 14 \leq x < 26\}$

______

PRACTICE: Express the given interval in set builder notation and graph.

$(-\infty, 0]$

______

TOPIC: LINEAR INEQUALITIES

PRACTICE: Express the given set in interval notation and graph.

$$\{x \mid x \leq 7\}$$

____

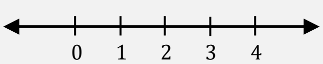
Solving Linear Inequalities

● Linear inequalities are just linear equations with an _____________ symbol instead of an = sign!

- You can apply ***everything*** from solving linear equations to linear inequalities.

Linear Equation

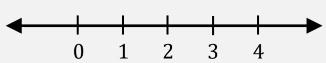
$$2x - \cancel{6} = 0$$
$$\cancel{+}6 \quad +6$$
$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$



Solution: ___________ value

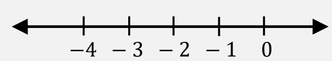
Linear Inequality

$$2x - 6 \leq 0$$



____

$$-2x - \cancel{6} \leq 0$$
$$\cancel{+}6 \quad +6$$
$$-2x \leq 6 \qquad -2x \leq 6$$



____

Multiplying/Dividing by a **negative** # _______ the symbol.

Solution: ___________ of values

PRACTICE: Solve the inequality. Express the solution set in interval notation and graph.

$$2x + 12 > 19$$

TOPIC: LINEAR INEQUALITIES
Fractions & Variables on Both Sides

● Solve *inequalities* with **fractions** and/or **variables on both sides** of the symbol just as you would a linear *equation*.

EXAMPLE**:** Solve the inequality. Express the solution set in interval notation and graph.

$$\frac{1}{4}(x+2) \geq -\frac{1}{12} - \frac{1}{3}x$$

PRACTICE**:** Solve the inequality. Express the solution set in interval notation and graph.

$$\frac{1}{3}(x+1) \geq \frac{1}{5}(3+2x)$$